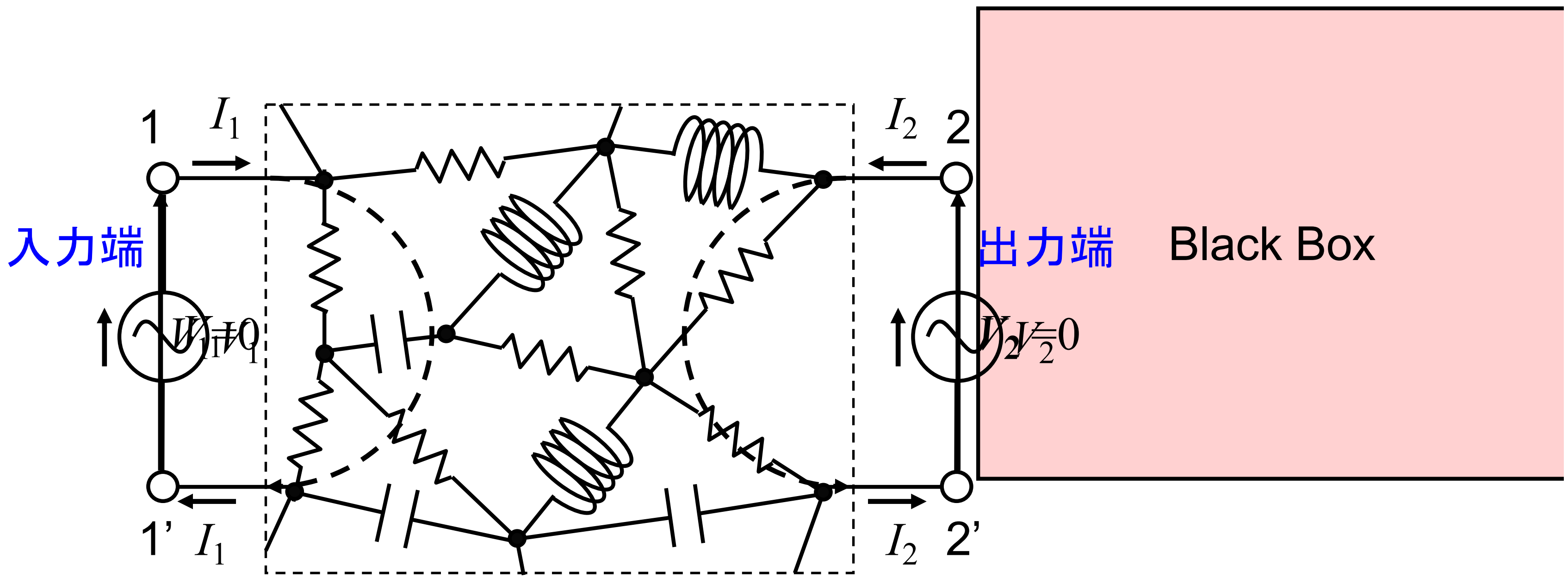
システムの表現

原因

結果

二端子対回路



内部には電源を含まないものとする

代表的な２端子対パラメータ

1. *Z*パラメータ（Z行列、インピーダンス行列）

$\dot{I}_1, \dot{I}_2$ をx_1, x_2 に、$\dot{V}_1, \dot{V}_2$ をy_1, y_2に割り当てると、回路の特性は次式で表される。

$$\begin{cases} \dot{V}_1 = \dot{Z}_{11}\dot{I}_1 + \dot{Z}_{12}\dot{I}_2 \\ \dot{V}_2 = \dot{Z}_{21}\dot{I}_1 + \dot{Z}_{22}\dot{I}_2 \end{cases}$$

$$a_{ij} \Rightarrow \dot{Z}_{ij},\ i, j = 1, 2$$

インピーダンス行列：Z行列

- 上式は次の行列表示の形で表せる。

$$\begin{bmatrix} \dot{V}_1 \\ \dot{V}_2 \end{bmatrix} = \begin{bmatrix} \dot{Z}_{11} & \dot{Z}_{12} \\ \dot{Z}_{21} & \dot{Z}_{22} \end{bmatrix} \begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \end{bmatrix}$$

Z行列

- その係数行列をZ 行列という。
  Z 行列の要素は Z パラメータという。

Zパラメータはいかに求めるか？

Zパラメータは次のように求められる。

$$\dot{Z}_{11} = \left.\frac{\dot{V}_1}{\dot{I}_1}\right|_{\dot{I}_2=0} =$$ 出力端開放 駆動点インピーダンス

$$\dot{Z}_{12} = \left.\frac{\dot{V}_1}{\dot{I}_2}\right|_{\dot{I}_1=0} =$$ 入力端開放 伝達インピーダンス

$$\dot{Z}_{21} = \left.\frac{\dot{V}_2}{\dot{I}_1}\right|_{\dot{I}_2=0} =$$ 出力端開放 伝達インピーダンス

$$\dot{Z}_{22} = \left.\frac{\dot{V}_2}{\dot{I}_2}\right|_{\dot{I}_1=0} =$$ 入力端開放 駆動点インピーダンス

駆動点側:同じ端子側の関係を表す場合;

伝達:入力側と出力側の関係を表す場合;

Zパラメータのインデックスの意味

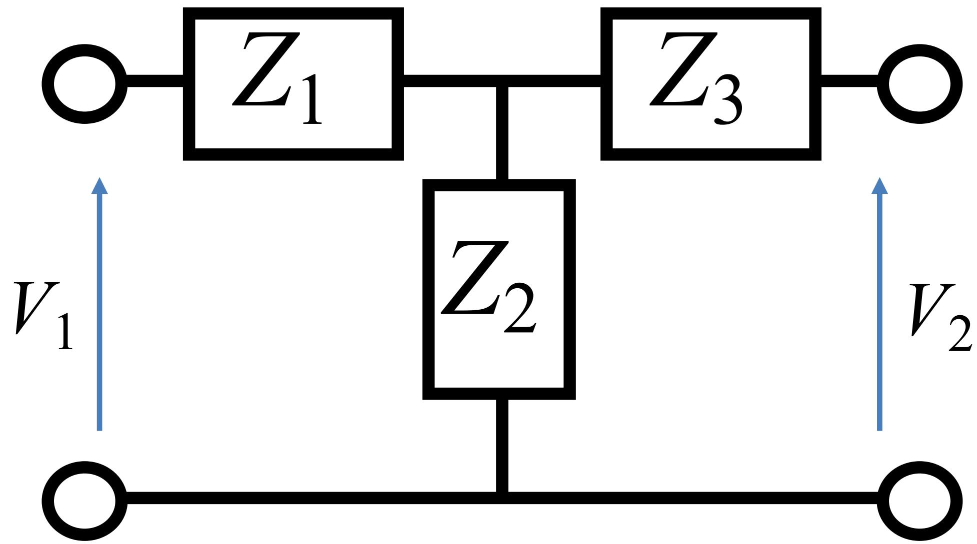
$$Z_{ij}$$

↓

$$Z_{ij} = \frac{V_i}{I_j}$$

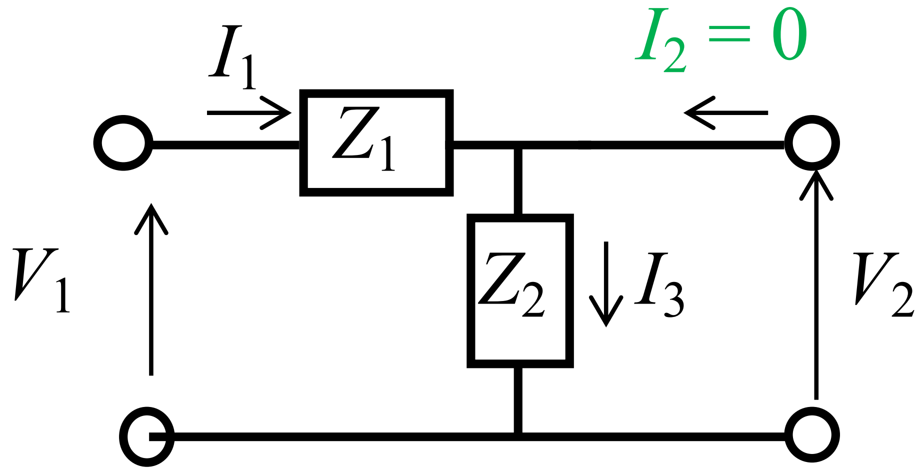
Z行列の導出

次のT型回路のZパラメータを求めよ。



Z行列の導出

まず、出力端開放($I_2 = 0$)で、電流 I_1 を流した場合、

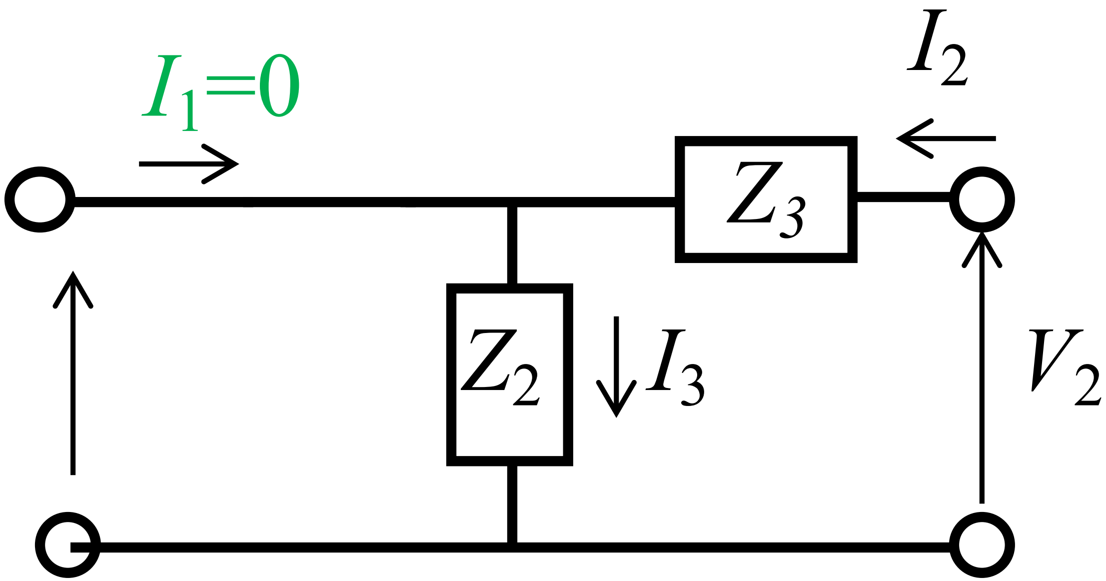


$$V_1 = (Z_1 + Z_2) I_1 \qquad \therefore \quad z_{11} = \frac{V_1}{I_1} = Z_1 + Z_2$$

$$V_2 = \frac{Z_2}{Z_1 + Z_2} V_1 = Z_2 I_1 \qquad \therefore \quad z_{21} = \frac{V_2}{I_1} = Z_2$$

Z行列の導出

次に、入力端開放($I_1 = 0$)で、電流 I_2 を流した場合、



$$V_1 = \frac{Z_2}{Z_2 + Z_3} V_2 = Z_2 I_2 \qquad \therefore \quad z_{12} = \frac{V_1}{I_2} = Z_2$$

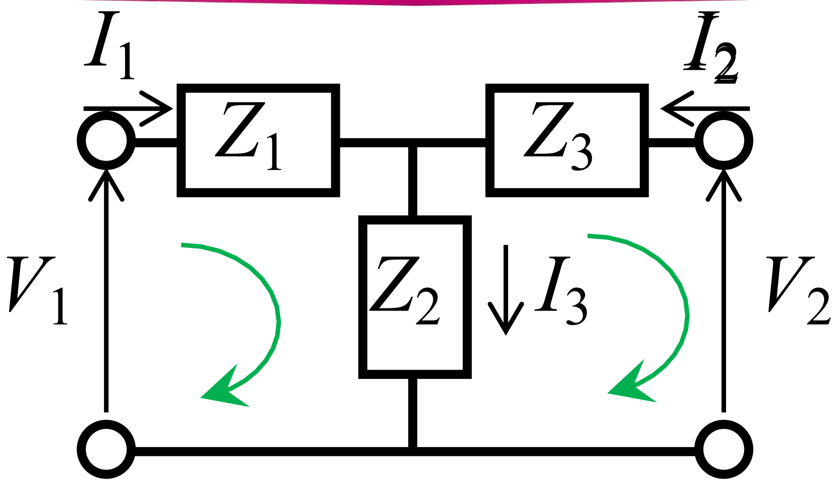
$$V_2 = (Z_2 + Z_3) I_2 \qquad \therefore \quad z_{22} = \frac{V_2}{I_2} = Z_2 + Z_3$$

Z行列の導出

従って、Z行列は、

$$
Z = \begin{bmatrix} Z_1 + Z_2 & Z_2 \\ Z_2 & Z_2 + Z_3 \end{bmatrix}
$$

Z行列の導出



T型回路

別の求め方として、Z_2 に流れる電流を図のようにI_3 と置くと、

$$I_3 = I_1 + I_2 \qquad \cdots(1)$$

$$V_1 = Z_1 I_1 + Z_2 I_3 \qquad \cdots(2)$$

$$V_2 = Z_3 I_2 + Z_2 I_3 \qquad \cdots(3)$$

Z行列の導出

式(1)を式(2), (3)に代入して整理すると、

$$V_1 = Z_1 I_1 + Z_2 \left(I_1 + I_2 \right) = \left(Z_1 + Z_2 \right) I_1 + Z_2 I_2 \quad \cdots (4)$$

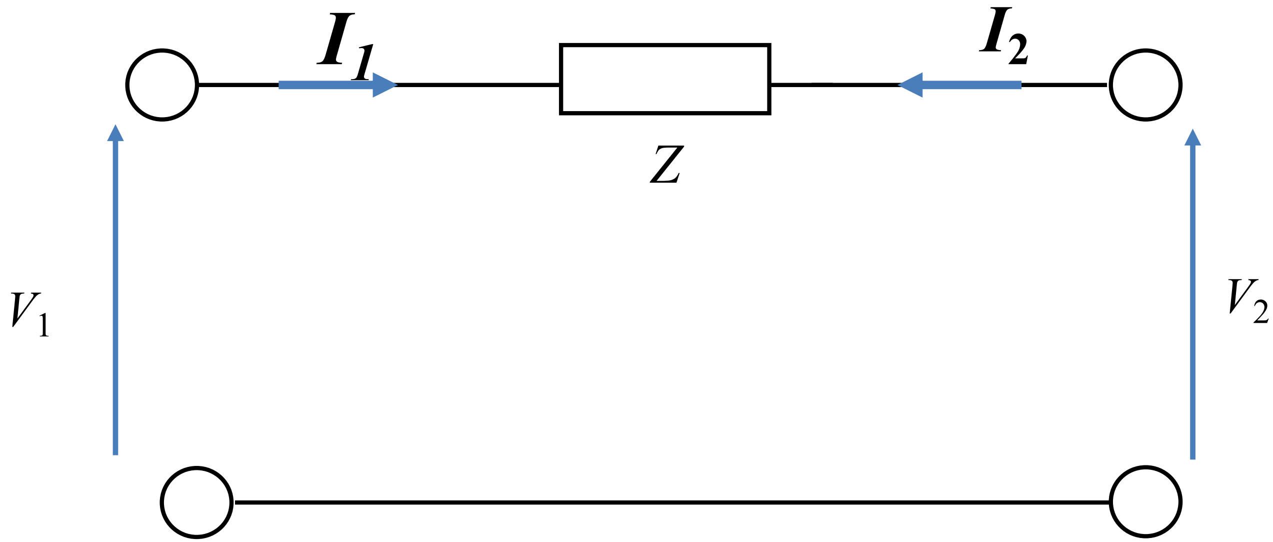
$$V_2 = Z_3 I_2 + Z_2 \left(I_1 + I_2 \right) = Z_2 I_1 + \left(Z_2 + Z_3 \right) I_2 \quad \cdots (5)$$

従って、Z行列は、

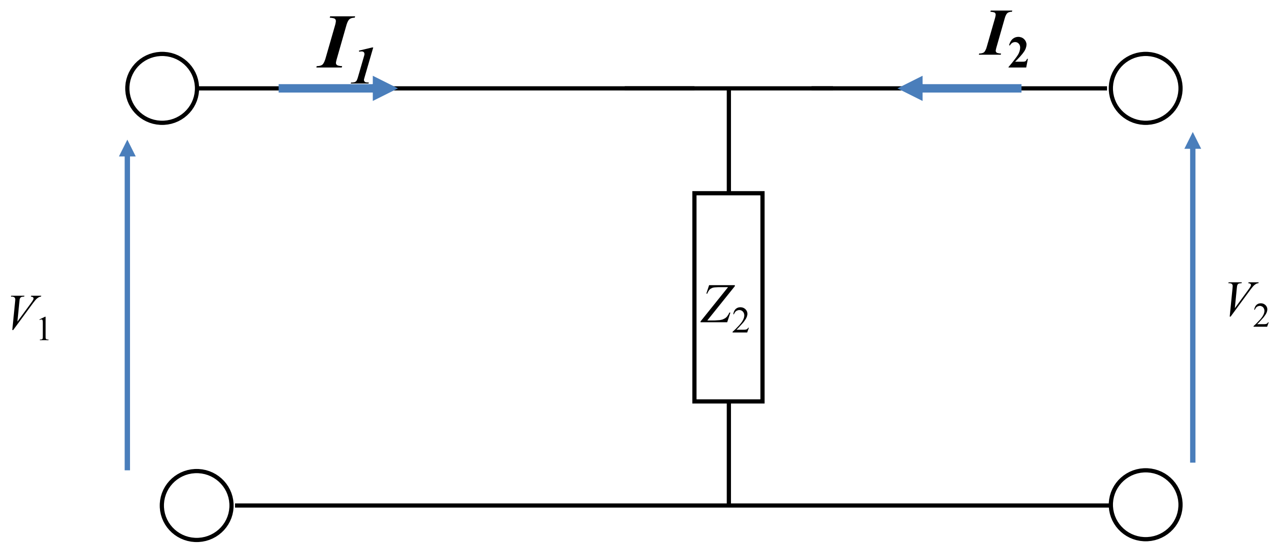
$$\mathrm{Z} = \begin{bmatrix} Z_1 + Z_2 & Z_2 \\ Z_2 & Z_2 + Z_3 \end{bmatrix}$$

練習：回路(a)(b)のZパラメータを求めよ

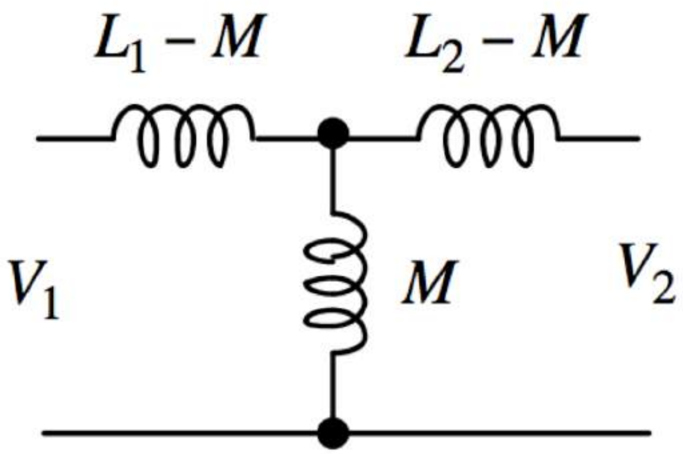
(a)

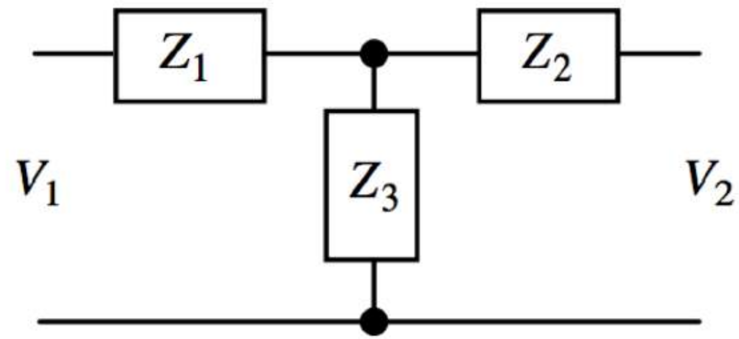


(b)



拡張:相互誘導回路におけるZ行列





$$\begin{pmatrix} Z'_{11} & Z'_{12} \\ Z'_{21} & Z'_{22} \end{pmatrix} = \begin{pmatrix} Z_1 + Z_3 & Z_3 \\ Z_3 & Z_2 + Z_3 \end{pmatrix}$$

$$\left.\begin{aligned} Z_1 &= j\omega(L_1 - M) \\ Z_2 &= j\omega(L_2 - M) \\ Z_3 &= j\omega M \end{aligned}\right\}$$

$$Z = \begin{bmatrix} jwL_1 & jwM \\ jwM & jwL_2 \end{bmatrix}$$

Yパラメータ/Y行列

$$\begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \end{bmatrix} = \begin{bmatrix} \dot{Y}_{11} & \dot{Y}_{12} \\ \dot{Y}_{21} & \dot{Y}_{22} \end{bmatrix} \begin{bmatrix} \dot{V}_1 \\ \dot{V}_2 \end{bmatrix}$$

Yパラメータ/Y行列：アドミタンス行列

$\dot{V}_1, \dot{V}_2$をx_1, x_2に、$\dot{I}_1, \dot{I}_2$をy_1, y_2に割り当てると、回路の特性は次式で表される。

$$\begin{cases} \dot{I}_1 = \dot{Y}_{11}\dot{V}_1 + \dot{Y}_{12}\dot{V}_2 \\ \dot{I}_2 = \dot{Y}_{21}\dot{V}_1 + \dot{Y}_{22}\dot{V}_2 \end{cases}$$

↓

$$\begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \end{bmatrix} = \begin{bmatrix} \dot{Y}_{11} & \dot{Y}_{12} \\ \dot{Y}_{21} & \dot{Y}_{22} \end{bmatrix} \begin{bmatrix} \dot{V}_1 \\ \dot{V}_2 \end{bmatrix}$$

Yパラメータ/Y行列

$$\dot{Y}_{11} = \left.\frac{\dot{I}_1}{\dot{V}_1}\right|_{\dot{V}_2=0}$$

出力端短絡　駆動点アドミタンス

$$\dot{Y}_{12} = \left.\frac{\dot{I}_1}{\dot{V}_2}\right|_{\dot{V}_1=0}$$

入力端短絡　伝達アドミタンス

$$\dot{Y}_{21} = \left.\frac{\dot{I}_2}{\dot{V}_1}\right|_{\dot{V}_2=0}$$

出力端短絡　伝達アドミタンス

$$\dot{Y}_{22} = \left.\frac{\dot{I}_2}{\dot{V}_2}\right|_{\dot{V}_1=0}$$

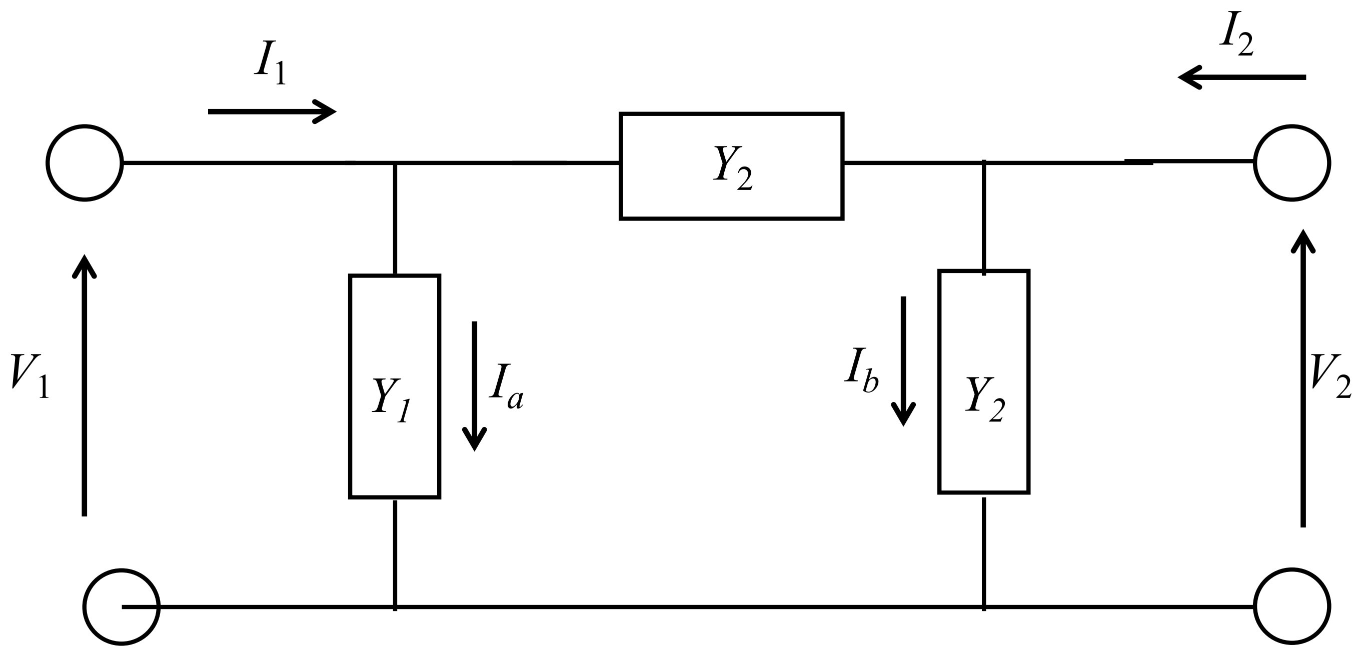
入力端短絡　駆動点アドミタンス

Yパラメータのインデックスの意味

$$Y_{ij}$$

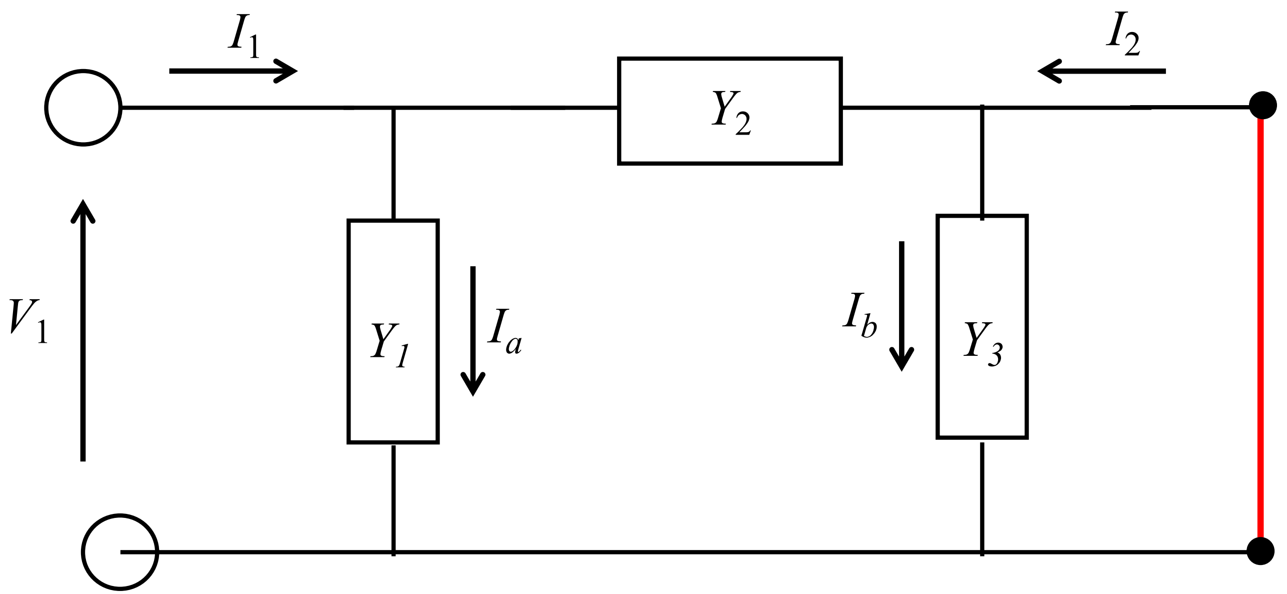
↓

$$Y_{ij} = \frac{I_i}{V_j}$$



π型回路

π型回路におけるYパラメータの算出



まず、出力端短絡($V_2 = 0$)で、V_1 を印加した場合、

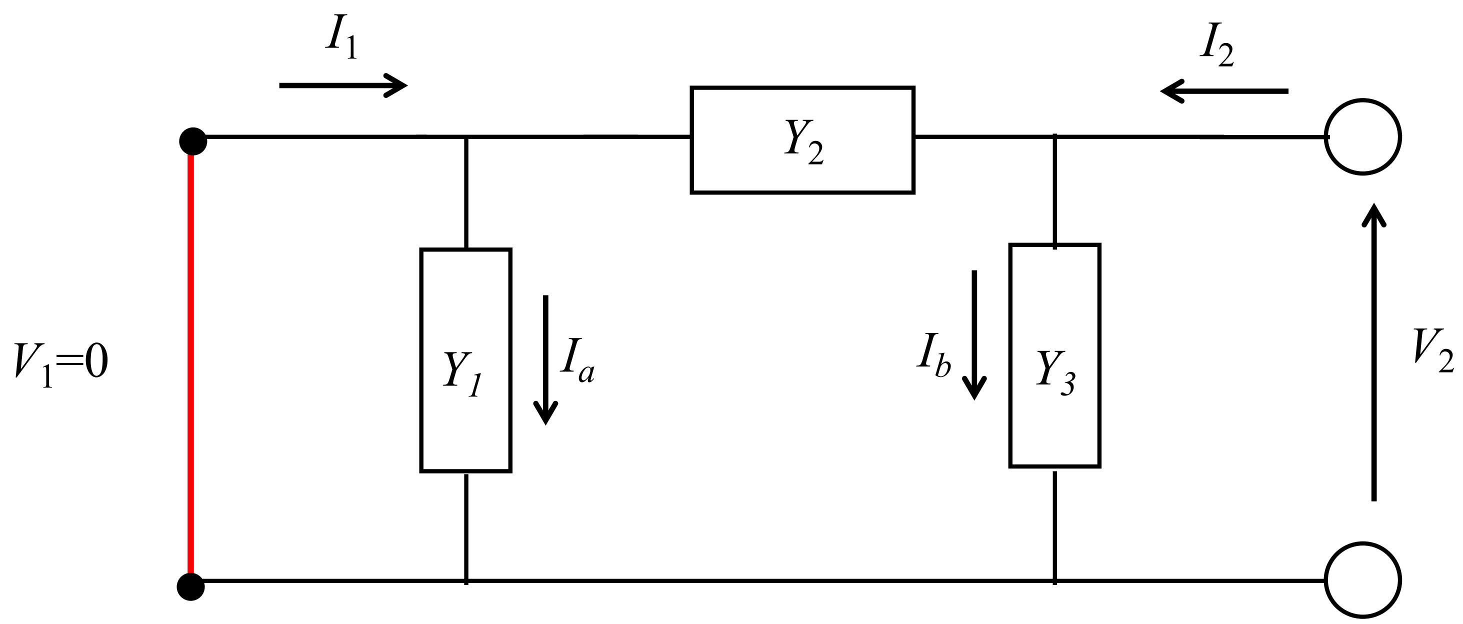
$$I_1 = (Y_1 + Y_2)V_1$$

$$Y_{11} = \frac{I_1}{V_1} = Y_1 + Y_2$$

$$I_2 = -\frac{Y_2}{Y_1 + Y_2} I_1 = -Y_2 V_1$$

$$Y_{21} = \frac{I_2}{V_1} = -Y_2$$

π型回路におけるYパラメータの算出



まず、出力端短絡($V_1 = 0$)で、V_1 を印加した場合、

$$I_2 = (Y_2 + Y_3)V_2 \qquad Y_{22} = \frac{I_2}{V_2} = Y_2 + Y_3$$

$$I_1 = -\frac{Y_2}{Y_2 + Y_3} I_2 = -Y_2 V_2 \qquad Y_{12} = \frac{I_2}{V_1} = -Y_2$$

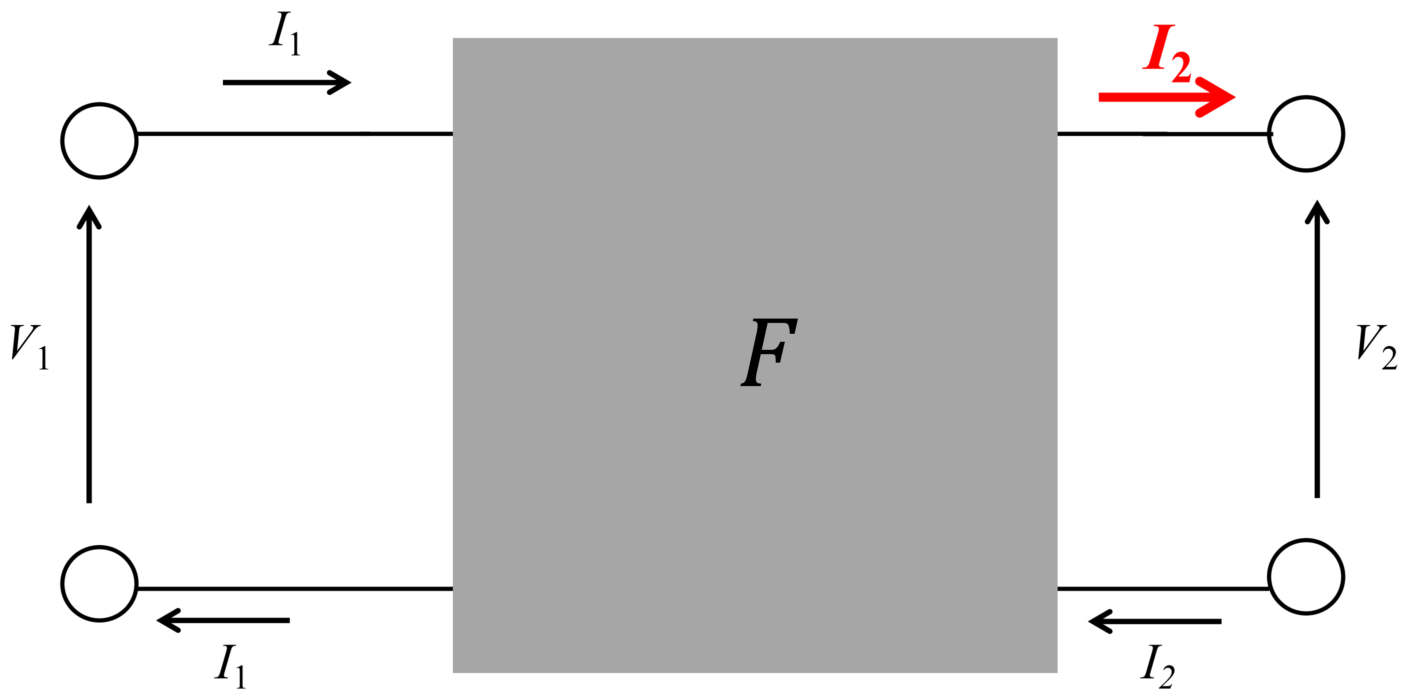
アドミタンス行列　Y行列

$$\mathrm{Y} = \begin{bmatrix} Y_1 + Y_2 & -Y_2 \\ -Y_2 & Y_2 + Y_3 \end{bmatrix}$$

F行列（K行列）

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

Fパラメータ　　F行列　　伝送行列



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

F行列の各要素の定義:

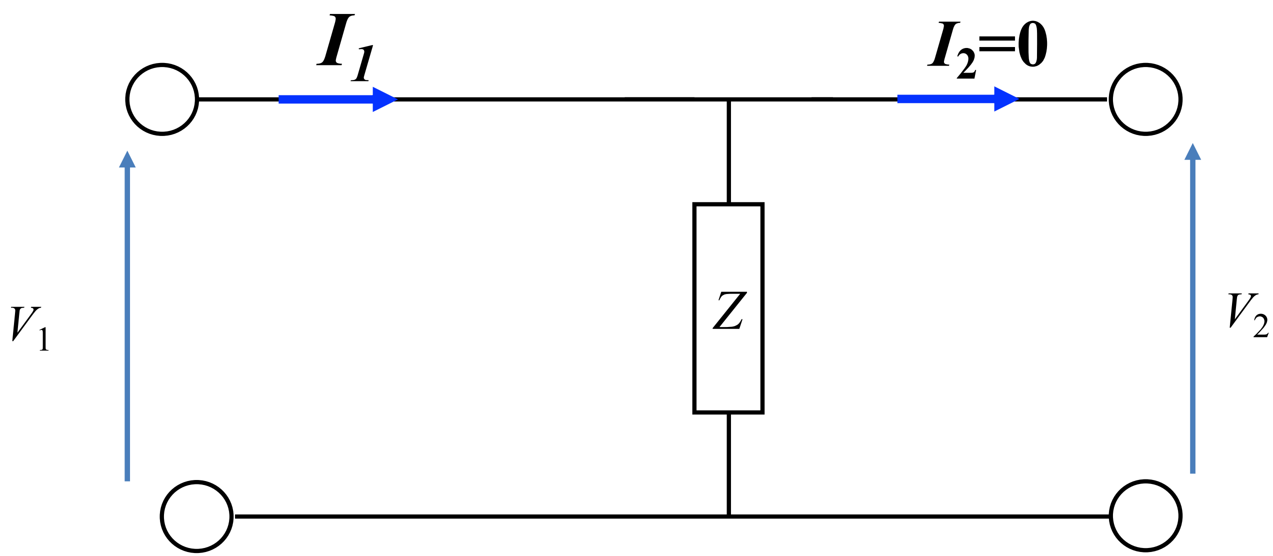
$$\dot{A} = \left.\frac{\dot{V}_1}{\dot{V}_2}\right|_{\dot{I}_2=0} = \text{出力端開放　電圧転送比}$$

$$\dot{B} = \left.\frac{\dot{V}_1}{\dot{I}_2}\right|_{\dot{V}_2=0} = \text{出力端短絡　伝達インピーダンス}$$

$$\dot{C} = \left.\frac{\dot{I}_1}{\dot{V}_2}\right|_{\dot{I}_2=0} = \text{出力端開放　伝達アドミタンス}$$

$$\dot{D} = \left.\frac{\dot{I}_1}{\dot{I}_2}\right|_{\dot{V}_2=0} = \text{出力端短絡　電流転送比}$$

F行列の各要素の導出：例1

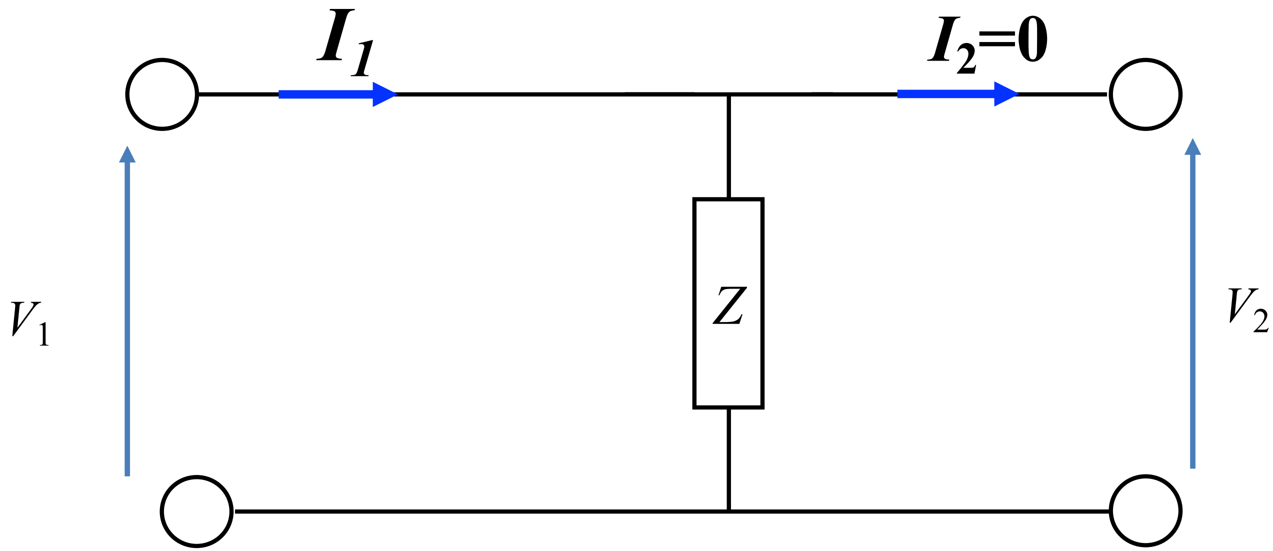


$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$$

$$I_2 = 0 \qquad V_1 = V_2$$

$$A = 1$$

F行列の各要素の導出：例1

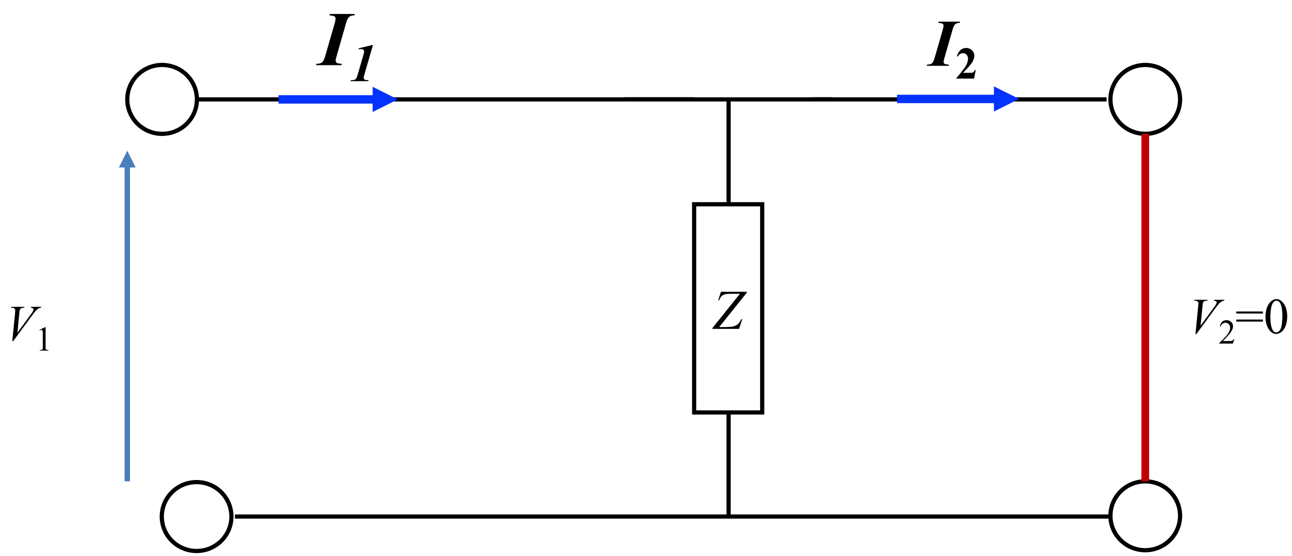


$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$$

$$V_1 = ZI_1 \qquad V_1 = V_2$$

$$C = \frac{1}{Z}$$

F行列の各要素の導出：例1

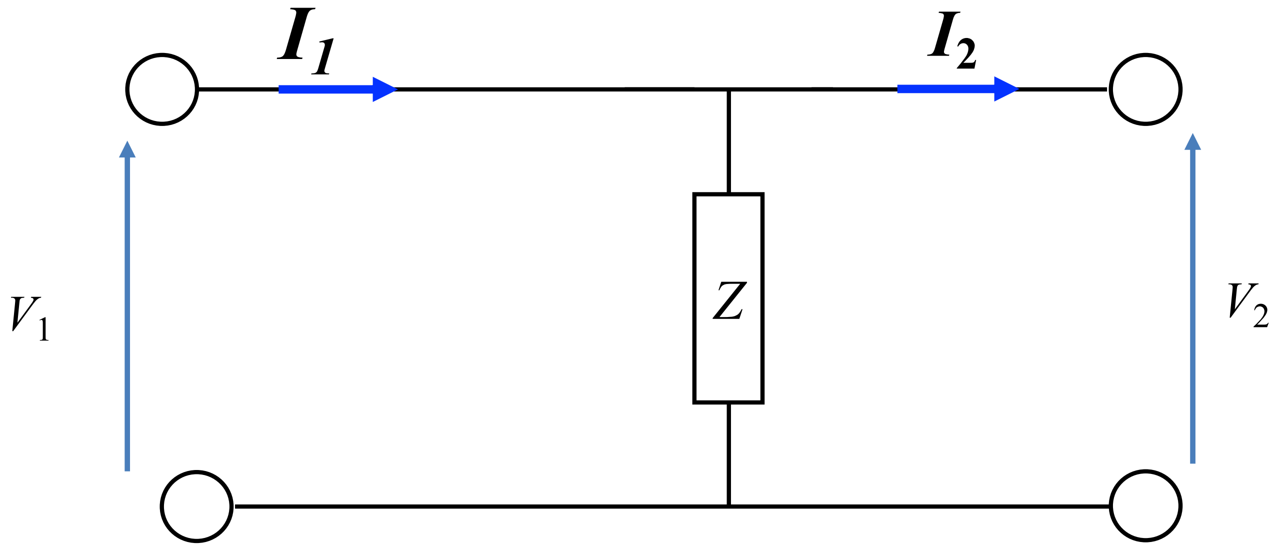


$$B = \left.\frac{V_1}{I_2}\right|_{V_2=0}$$

$$V_1 = V_2 = 0$$

$$B = 0$$

F行列の各要素の導出：例1

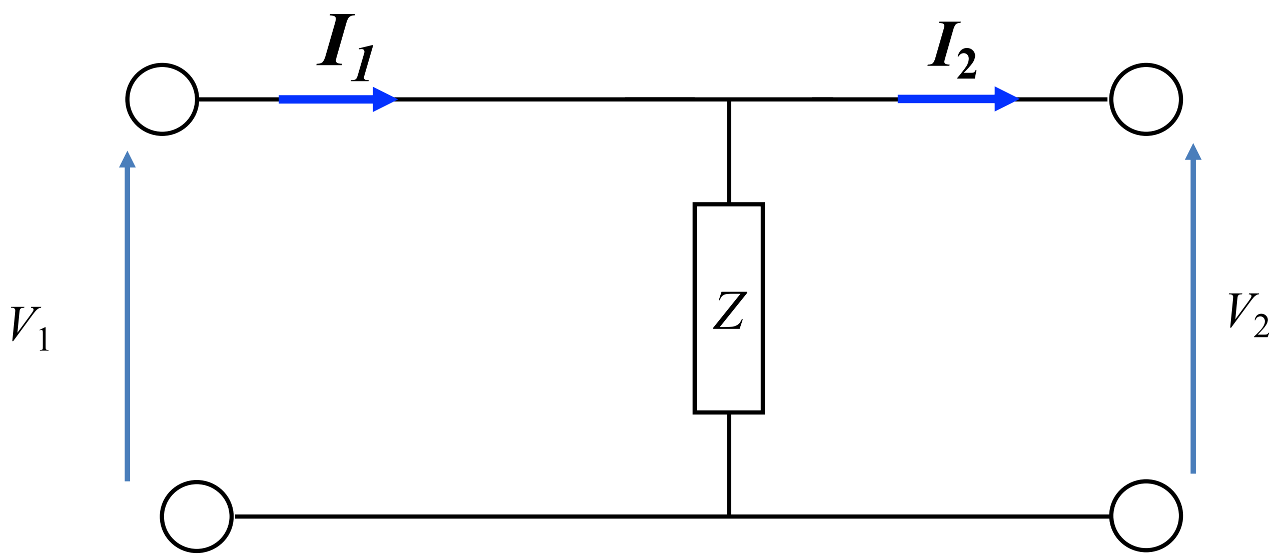


$$D = \left.\frac{I_1}{I_2}\right|_{V_2=0}$$

$$I_1 = I_2$$

$$D = 1$$

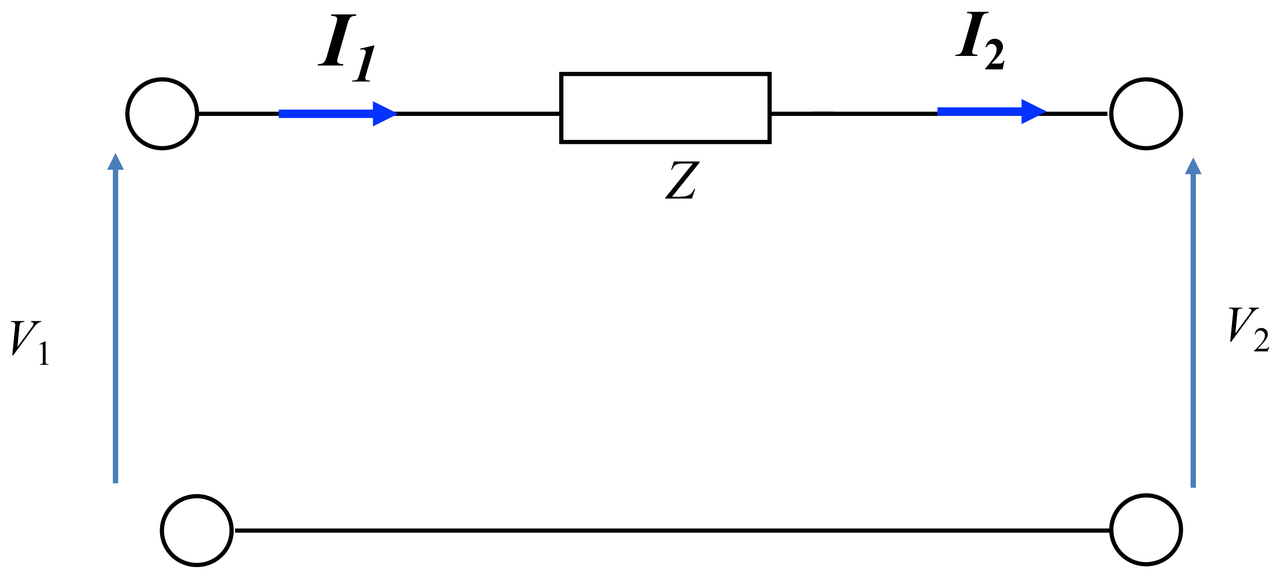
F行列の各要素の導出:例1



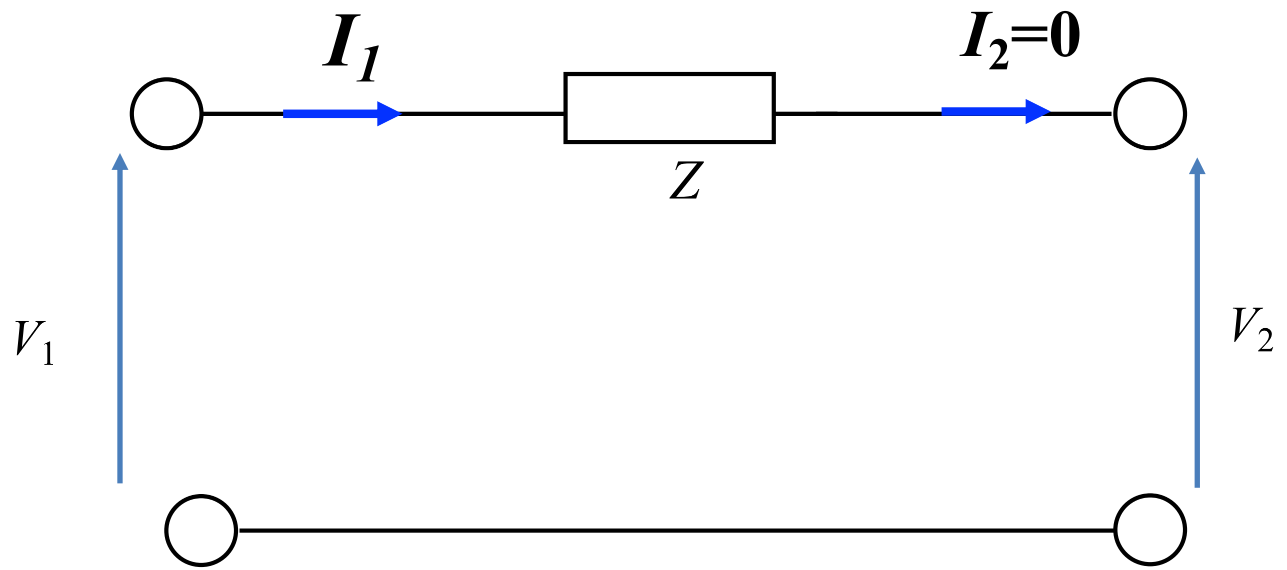
$A = 1$ $B = 0$ $C = \dfrac{1}{Z}$ $D = 1$

$$[F] = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1/z & 1 \end{bmatrix}$$

F行列の各要素の導出：例2



F行列の各要素の導出：例2

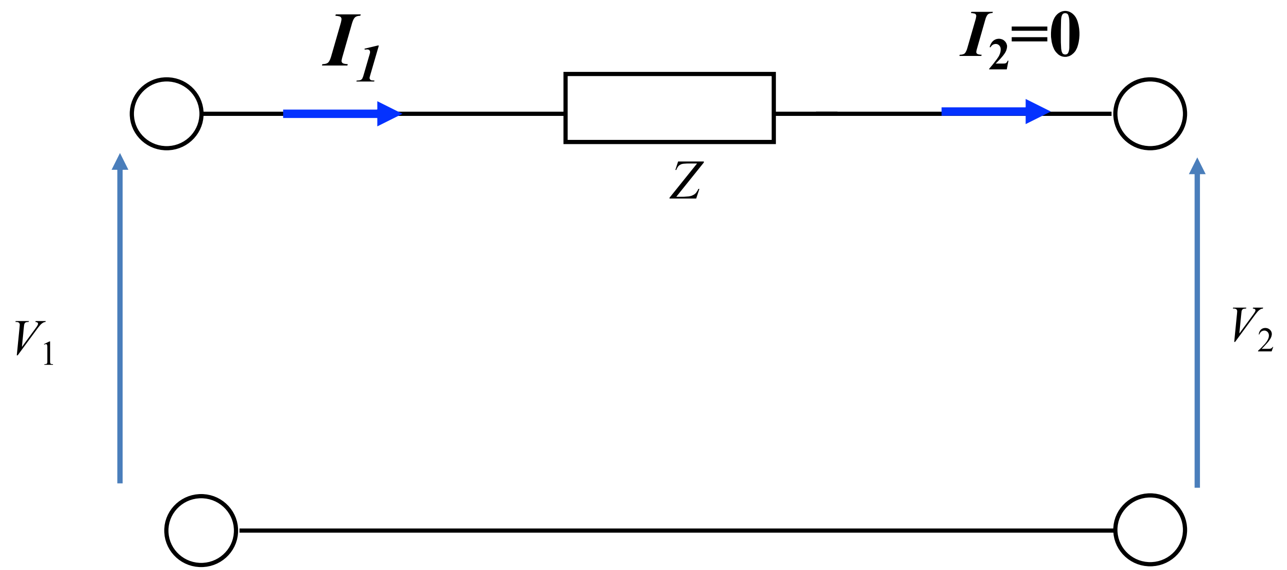


$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$$

$$I_2 = 0 \qquad V_1 = V_2$$

$$A = 1$$

F行列の各要素の導出：例2

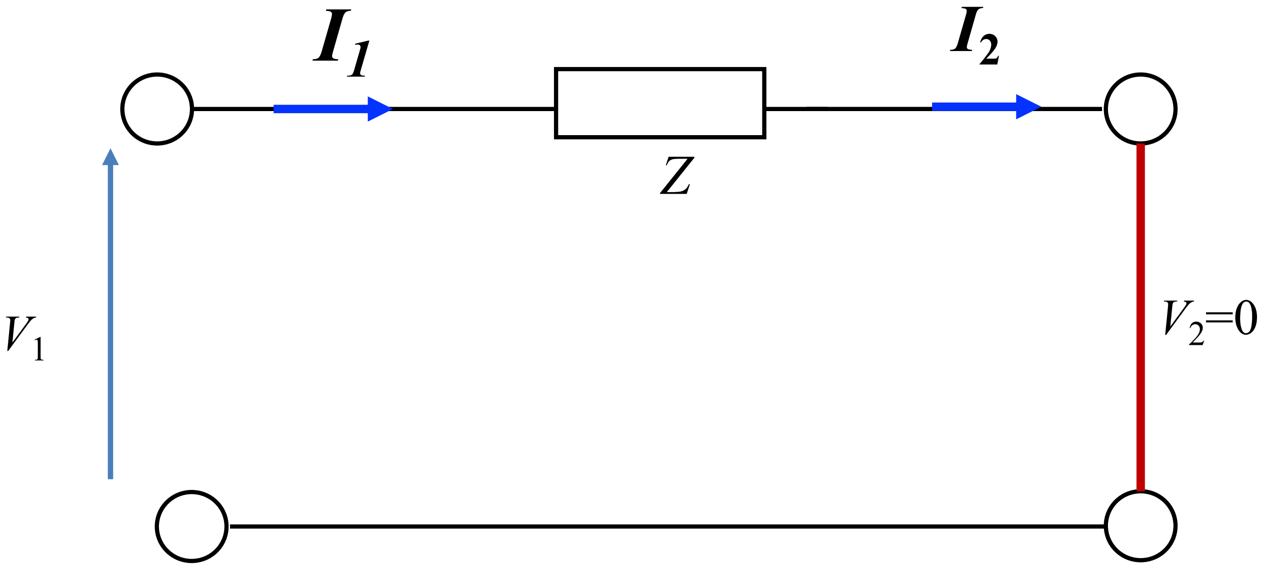


$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$$

$$I_1 = 0$$

$$C = 0$$

F行列の各要素の導出：例2

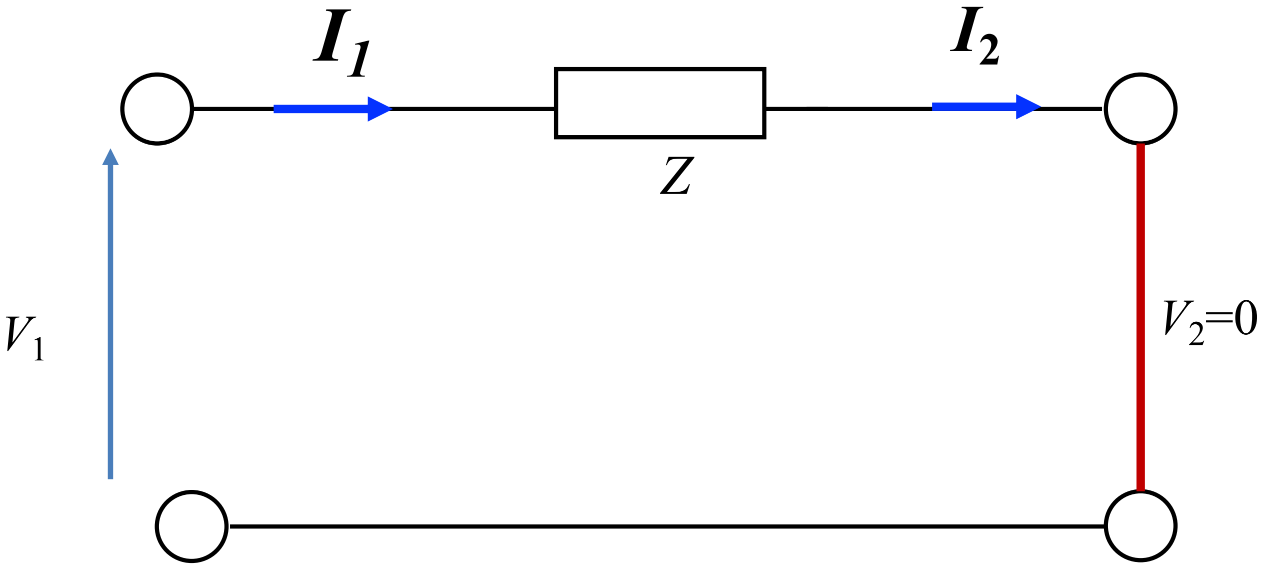


$$B = \left.\frac{V_1}{I_2}\right|_{V_2=0}$$

$$I_1 = I_2 \qquad V_1 = I_2 Z$$

$$B = Z$$

F行列の各要素の導出：例2

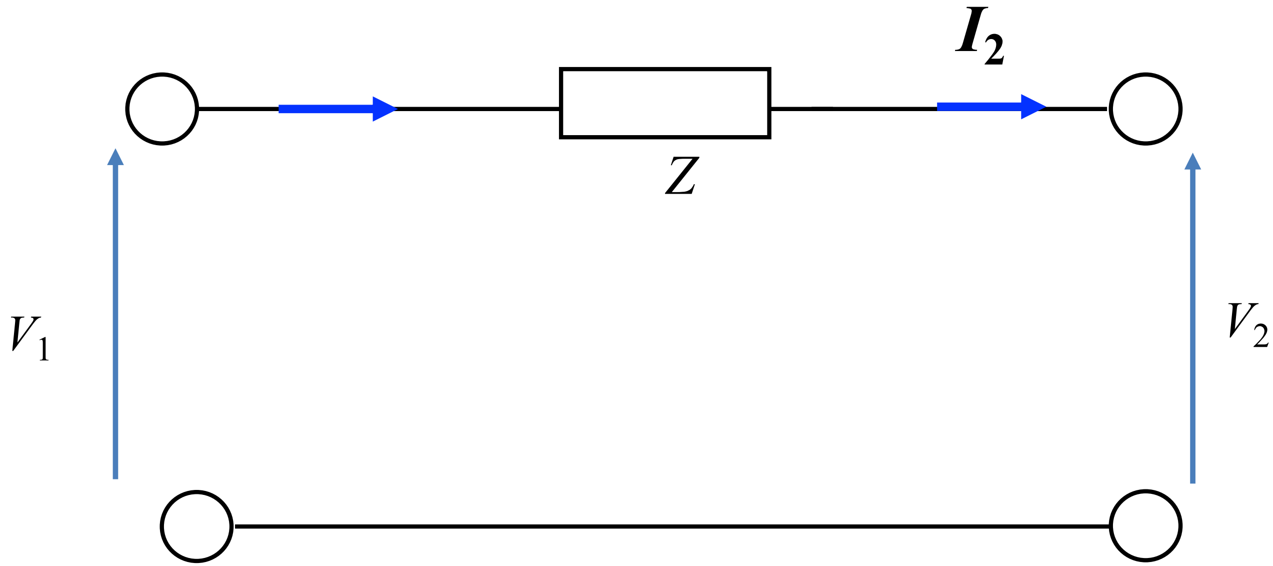


$$D = \left.\frac{I_1}{I_2}\right|_{V_2=0}$$

$$I_1 = I_2$$

$$D = 1$$

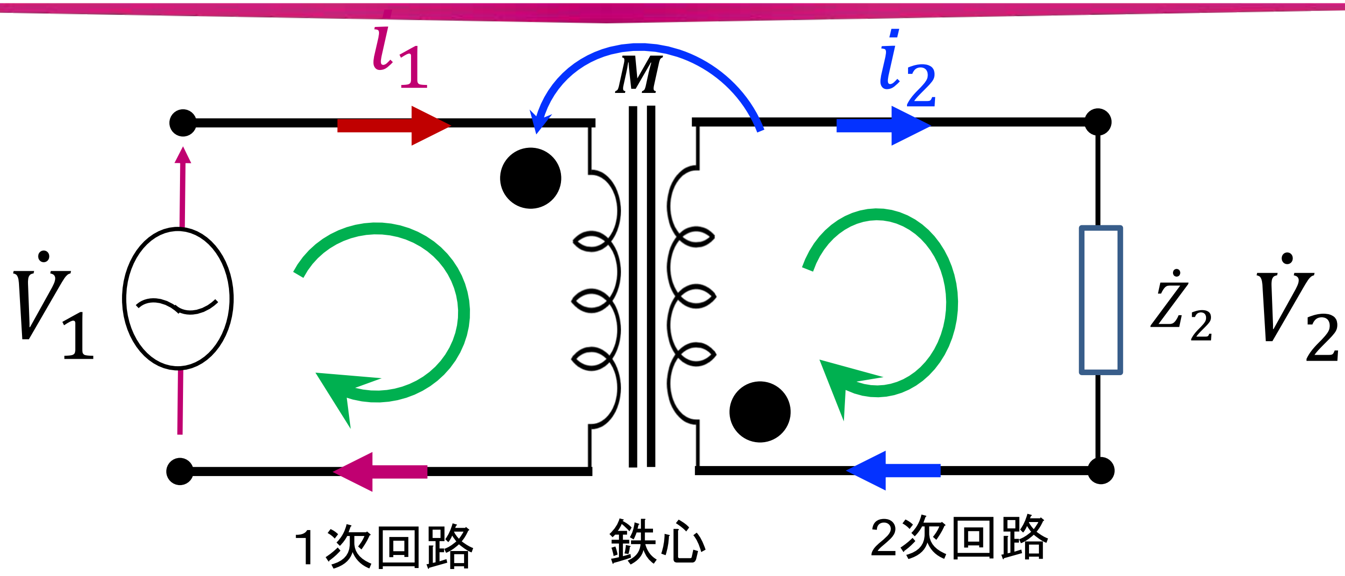
F行列の各要素の導出：例2



$$A = 1 \qquad B = Z \qquad C = 0 \qquad D = 1$$

$$[F] = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$$

拡張:変圧器結合回路:電流関係　理想変圧器



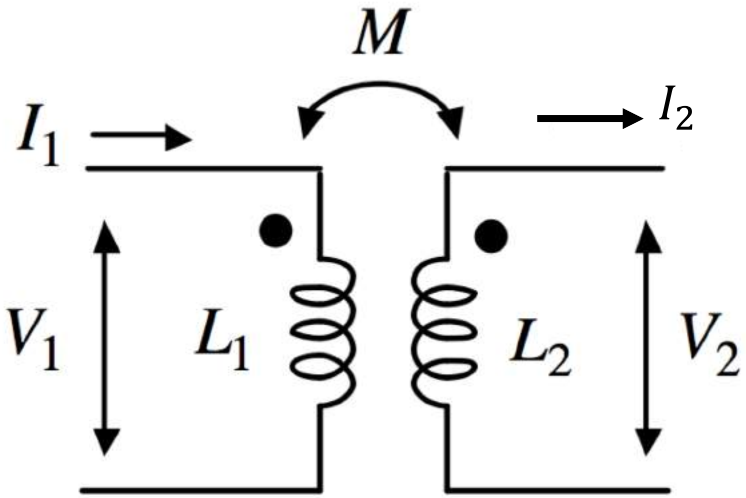
1次側のコイルの巻き数は2次側のコイルの巻き数の n倍である

$$\dot{Z}_1 = n^2 \dot{Z}_2$$

$$i_2 = \frac{\dot{V}_2}{\dot{Z}_2} = \frac{\frac{1}{n}\dot{V}_1}{\dot{Z}_2} = \frac{\frac{1}{n}\dot{V}_1}{\frac{1}{n^2}\dot{Z}_1} = n\, i_1$$

$$\dot{V}_1 = n\, \dot{V}_2$$

図に示した相互誘導回路は変圧器と呼ばれ、交流電源の電圧と電流の大きさを変換する機能を持つ。2次側のコイルの巻き数は1次側のコイルの巻き数の n倍である このとき、理想的な変圧器では、2次側電圧は1次側のn倍になり、電流は$1/n$になる。この変圧器に対するF行列をもとめよ。



$$V_2 = nV_1$$

$$I_2 = I_1/n$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$$V_1 = AV_2 + BI_2 \qquad I_1 = CV_2 + DI_2$$

題意より、

$$V_2 = nV_1 \qquad I_2 = I_1/n \qquad V_1 = \frac{1}{n}V_2 \qquad I_1 = nI_2$$

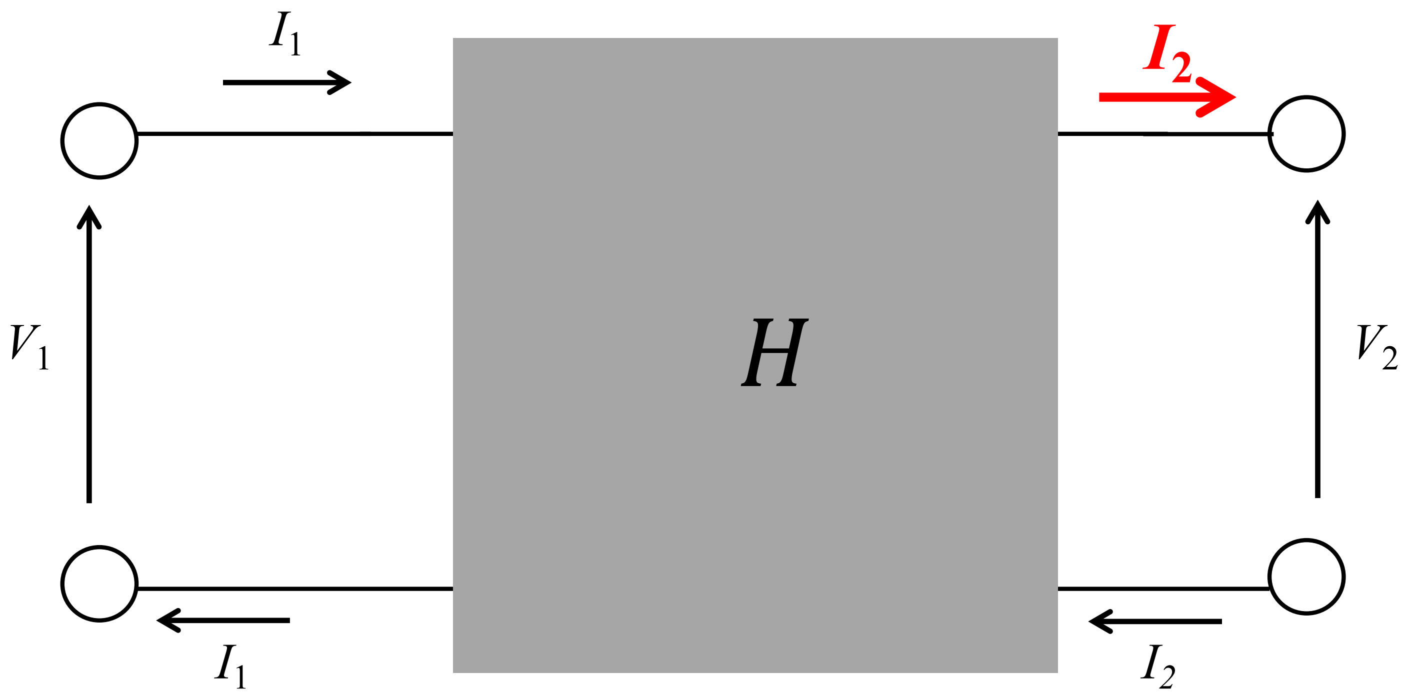
$$A = \frac{1}{n} \qquad B = 0 \qquad C = 0 \qquad D = n$$

$$F = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{1}{n} & 0 \\ 0 & n \end{bmatrix}$$

Hパラメータ　　H行列

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} V_2 \\ I_1 \end{bmatrix}$$

Hパラメータ　H行列　ハイブリッド行列



$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} V_2 \\ I_1 \end{bmatrix}$$

H行列の各要素の定義:

$$H_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} =$$ 出力端短絡　駆動点インピーダンス

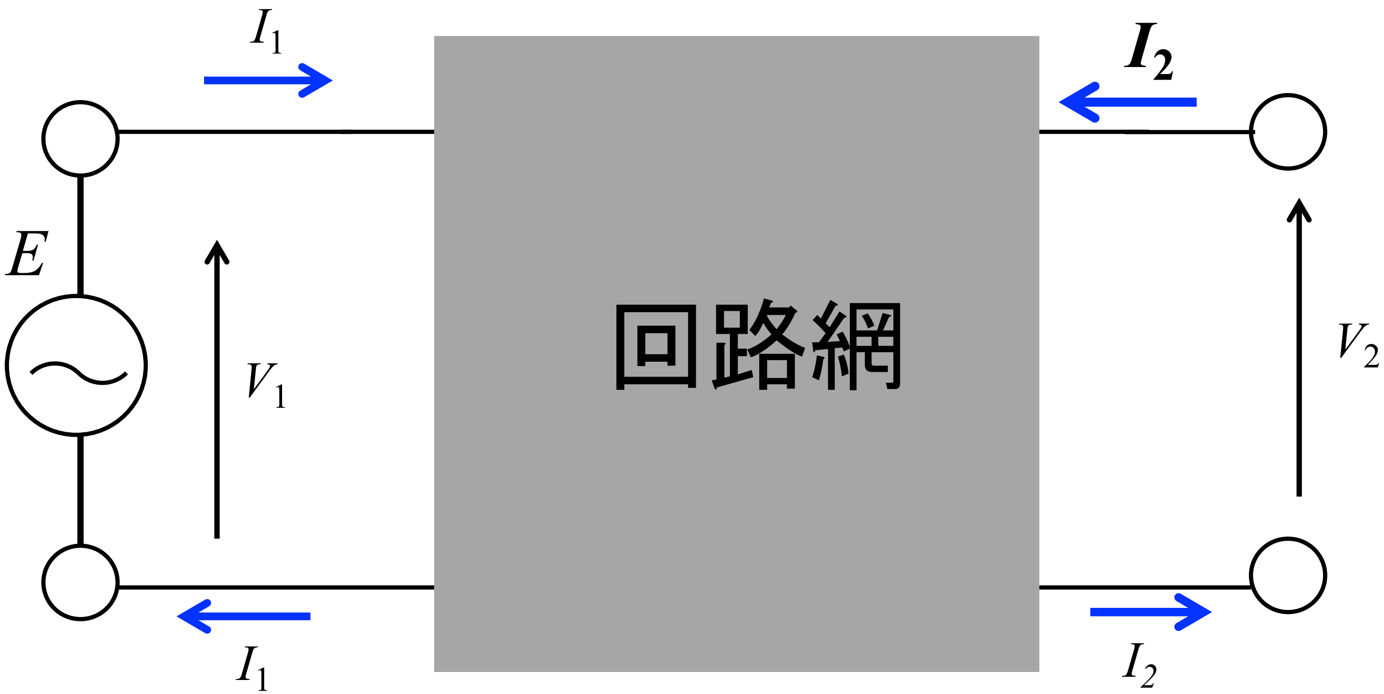
$$H_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} =$$ 入力端開放　電圧伝達比

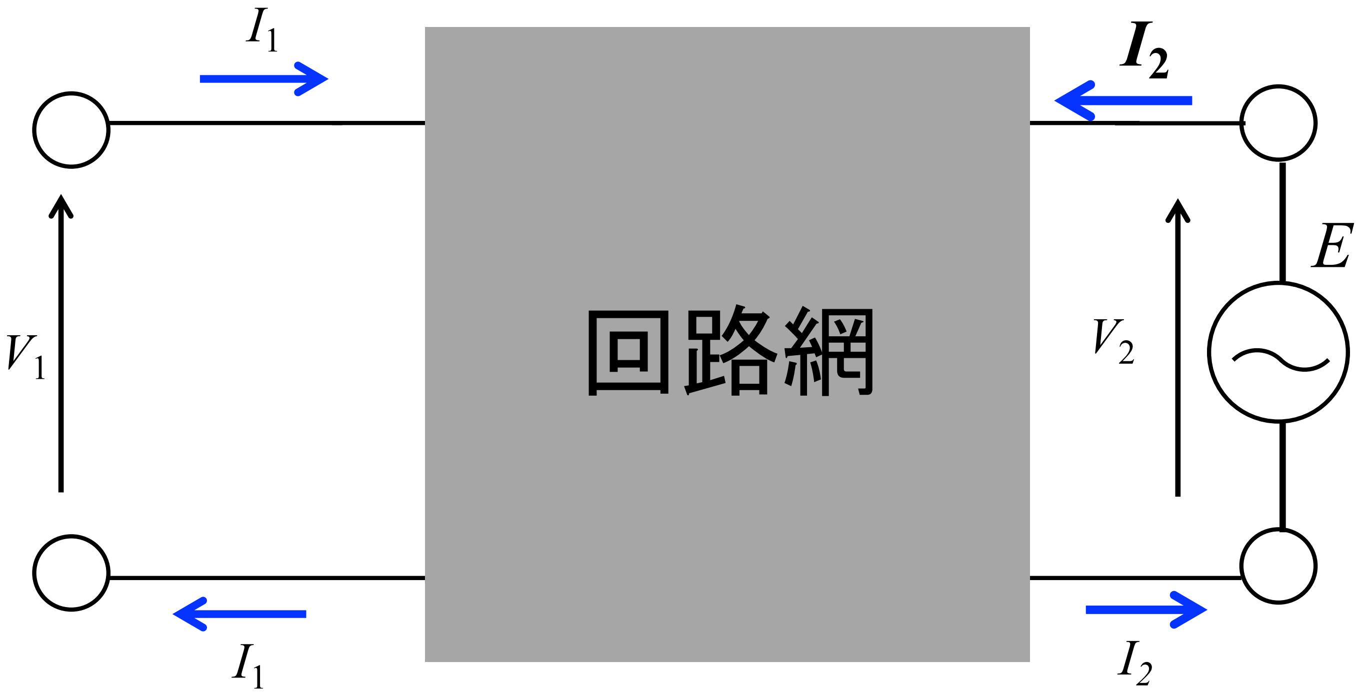
$$H_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} =$$ 出力端短絡　電流伝達比

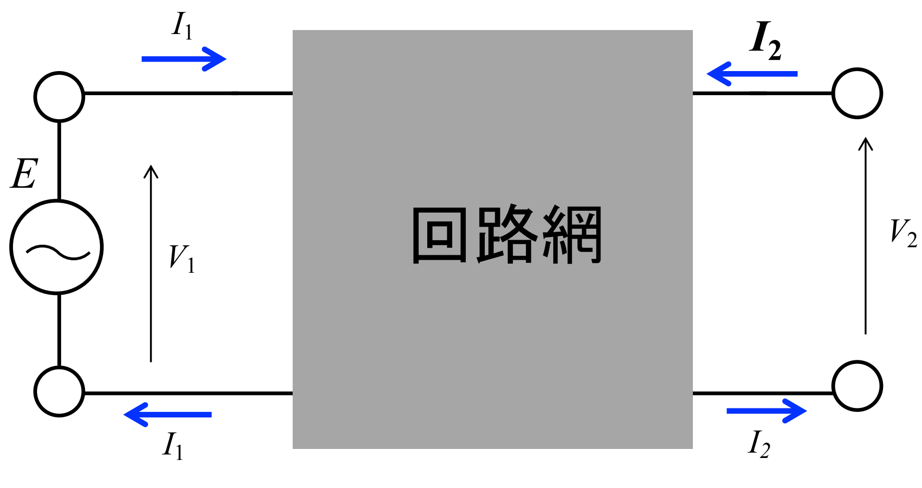
$$H_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} =$$ 入力端開放　駆動点アドミタンス

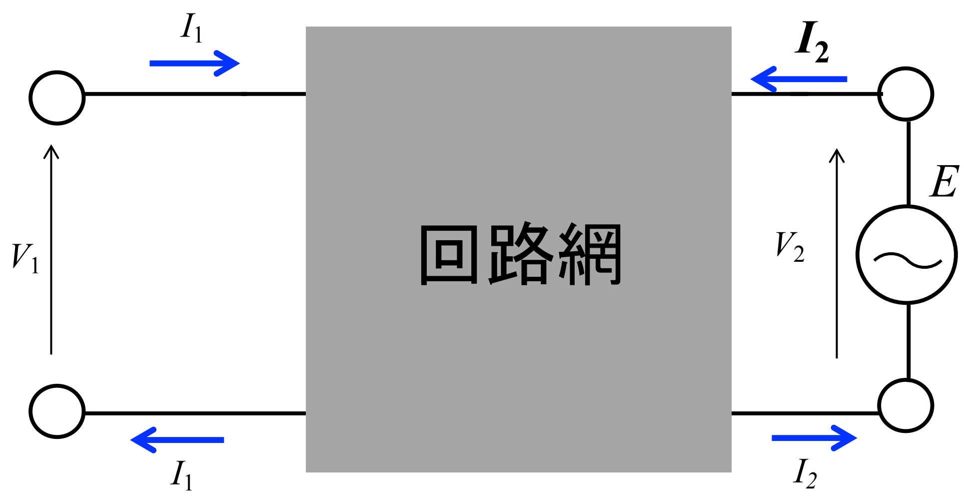
2端子対回路の行列変換

相反と対称




I_1
I_2
E
V_1
回路網
V_2
I_1
I_2





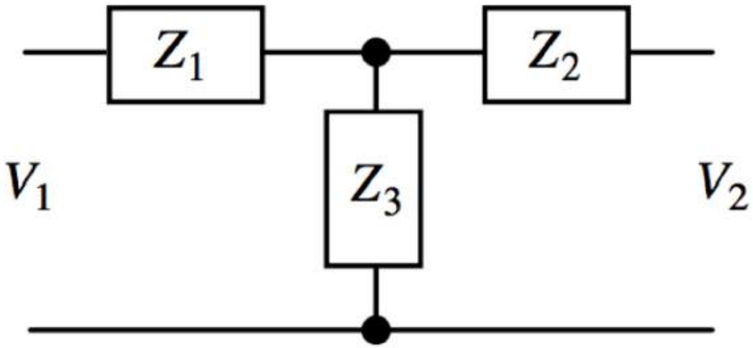
$$I_1 = I_2$$

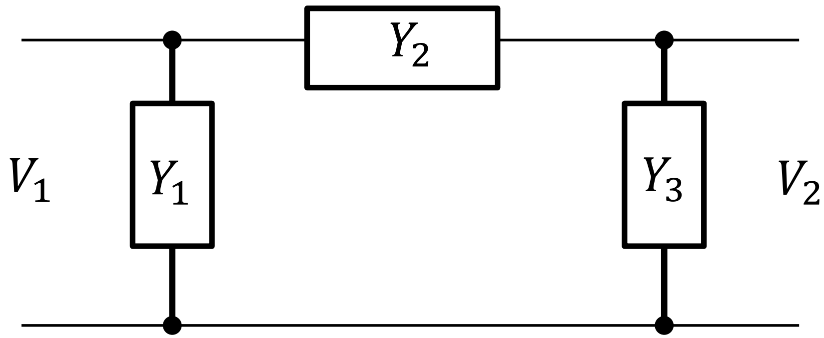
$$Z_{12} = Z_{21} \qquad Y_{12} = Y_{21}$$

R、L、Cのような受動線形素子のみから校正されている回路では、**相反性**は常に成り立つ

対称回路

T型回路やπ型回路のように、左右対称な構造を持つ場合には、この回路は対称性があるといい、以下の性質を持つ。





$$Z_{11} = Z_{22}$$

$$Y_{11} = Y_{22}$$

諸行列の関係

Y行列とZ行列との関係

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$[I] = [Y][V]$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$[V] = [Z][I]$$

$$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} y_{22} & -y_{12} \\ -y_{21} & y_{11} \end{bmatrix}$$

$$\Delta = y_{11}y_{22} - y_{12}y_{21}$$

$$[Z] = [Y]^{-1}$$

逆行列の関係式を用いると:

$$[Z] = [Y]^{-1}$$

$$[Y]^{-1} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}^{-1}$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}^{-1} = \frac{1}{Y_{22}Y_{11} - Y_{21}Y_{12}} \begin{bmatrix} Y_{22} & -Y_{12} \\ -Y_{21} & Y_{11} \end{bmatrix}$$

$$[Z] = \frac{1}{Y_{22}Y_{11} - Y_{21}Y_{12}} \begin{bmatrix} Y_{22} & -Y_{12} \\ -Y_{21} & Y_{11} \end{bmatrix}$$

Y行列とZ行列との関係：連立方程式

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} Z_{11}Y_{11} + Z_{21}Y_{12} & Z_{12}Y_{11} + Z_{22}Y_{12} \\ Z_{11}Y_{21} + Z_{21}Y_{22} & Z_{12}Y_{21} + Z_{22}Y_{22} \end{bmatrix}$$

$$\begin{cases} Z_{11}Y_{11} + Z_{21}Y_{12} = 1 \\ Z_{12}Y_{11} + Z_{22}Y_{12} = 0 \\ Z_{11}Y_{21} + Z_{21}Y_{22} = 0 \\ Z_{12}Y_{21} + Z_{22}Y_{22} = 1 \end{cases}$$

$$\begin{cases} Z_{11}Y_{11} + Z_{21}Y_{12} = 1 \\ Z_{12}Y_{11} + Z_{22}Y_{12} = 0 \\ Z_{11}Y_{21} + Z_{21}Y_{22} = 0 \\ Z_{12}Y_{21} + Z_{22}Y_{22} = 1 \end{cases}$$

Y行列とZ行列との関係

$$\begin{cases} Z_{11}Y_{11} + Z_{21}Y_{12} = 1 \\ Z_{12}Y_{11} = -Z_{22}Y_{12} \\ Z_{11}Y_{21} = -Z_{21}Y_{22} \\ Z_{12}Y_{21} + Z_{22}Y_{22} = 1 \end{cases}$$

$$\begin{cases} Z_{21}\dfrac{-Y_{22}}{Y_{21}}Y_{11} + Z_{21}Y_{12} = 1 \\ \dfrac{-Y_{12}}{Y_{11}}Z_{22}Y_{21} + Z_{22}Y_{22} = 1 \end{cases}$$

$$\begin{cases} Z_{11}Y_{11} + Z_{21}Y_{12} = 1 \\ Z_{12} = \dfrac{-Y_{12}}{Y_{11}}Z_{22} \\ Z_{11} = \dfrac{-Y_{22}}{Y_{21}}Z_{21} \\ Z_{12}Y_{21} + Z_{22}Y_{22} = 1 \end{cases}$$

$$\begin{cases} Z_{21} = \dfrac{-Y_{21}}{Y_{22}Y_{11} - Y_{21}Y_{12}} \\ Z_{22} = \dfrac{Y_{11}}{Y_{22}Y_{11} - Y_{21}Y_{12}} \end{cases}$$

$$\begin{cases} Z_{12} = \dfrac{-Y_{12}}{Y_{22}Y_{11} - Y_{21}Y_{12}} \\ Z_{11} = \dfrac{Y_{22}}{Y_{21}Y_{12} - Y_{22}Y_{11}} \end{cases}$$

Y行列とZ行列との関係

$$\begin{cases} Z_{12} = \dfrac{-Y_{12}}{Y_{21}Y_{12} - Y_{22}Y_{11}} \\ Z_{11} = \dfrac{Y_{22}}{Y_{21}Y_{12} - Y_{22}Y_{11}} \end{cases}$$

$$\begin{cases} Z_{21} = \dfrac{-Y_{21}}{Y_{21}Y_{12} - Y_{22}Y_{11}} \\ Z_{22} = \dfrac{Y_{11}}{Y_{21}Y_{12} - Y_{22}Y_{11}} \end{cases}$$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \frac{1}{Y_{22}Y_{11} - Y_{21}Y_{12}} \begin{bmatrix} Y_{22} & -Y_{12} \\ -Y_{21} & Y_{11} \end{bmatrix}$$

$$\Delta = Y_{22}Y_{11} - Y_{21}Y_{12}$$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} Y_{22} & -Y_{12} \\ -Y_{21} & Y_{11} \end{bmatrix}$$

証明終了

Y行列とZ行列: 逆行列の関係

Y行列➡Z行列:

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \frac{1}{\Delta_Y} \begin{bmatrix} Y_{22} & -Y_{12} \\ -Y_{21} & Y_{11} \end{bmatrix}$$

$$\Delta = \boldsymbol{Y_{22}Y_{11} - Y_{21}Y_{12}}$$

Z行列➡Y行列:

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \frac{1}{\Delta_Z} \begin{bmatrix} Z_{22} & -Z_{12} \\ -Z_{21} & Z_{11} \end{bmatrix}$$

$$\Delta = \boldsymbol{Z_{22}Z_{11} - Z_{21}Z_{12}}$$

Z行列 → F行列の変換

$$[F] = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

$$[F] = \frac{1}{Z_{21}} \begin{bmatrix} Z_{21} & Z_{22}Z_{11} - Z_{21}Z_{12} \\ 1 & Z_{22} \end{bmatrix}$$

F行列 → Z行列の変換

$$[Z] = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

$$[Z] = \begin{bmatrix} A/C & 1/C \\ 1/C & D/C \end{bmatrix}$$

理想変圧器のZ行列

$$[Z] = \begin{bmatrix} A/C & 1/C \\ 1/C & D/C \end{bmatrix}$$

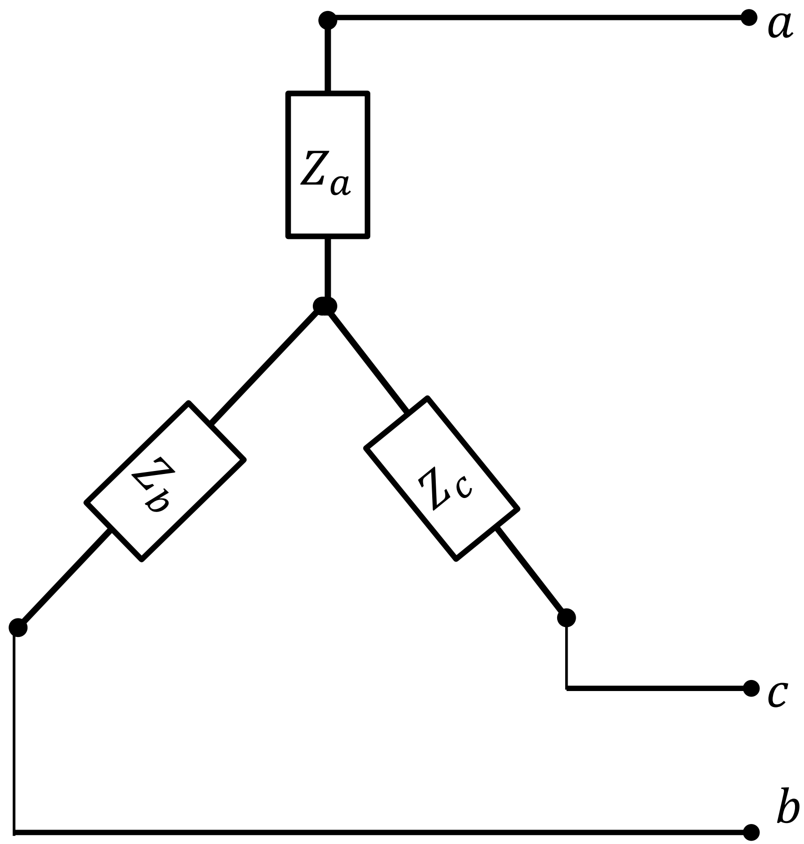
$$F = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{1}{n} & 0 \\ 0 & n \end{bmatrix}$$

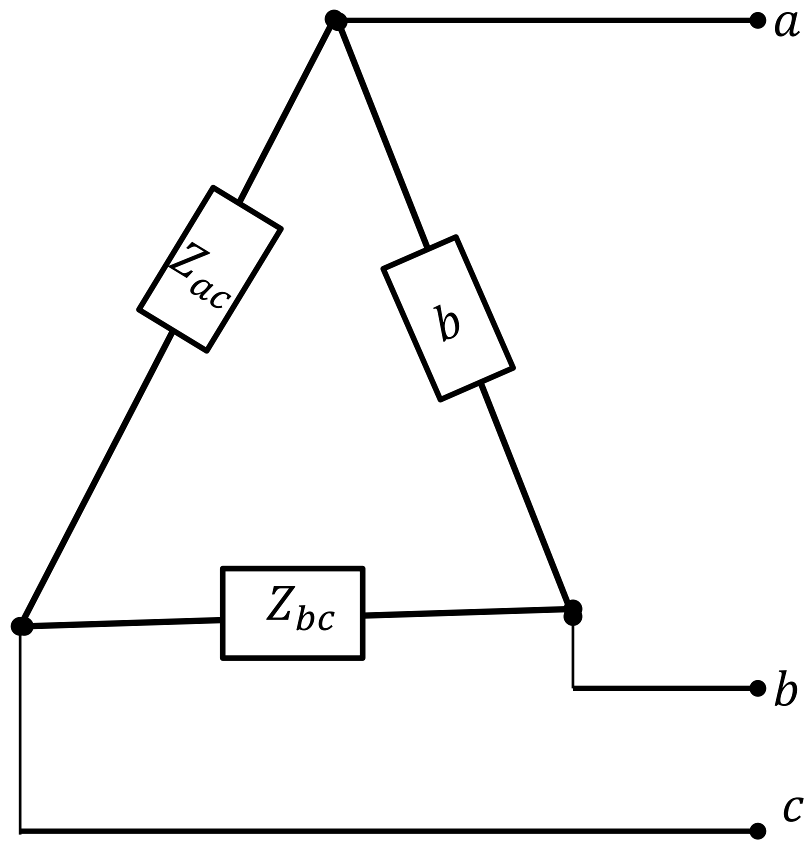
$$[Z]$$

$$\downarrow$$

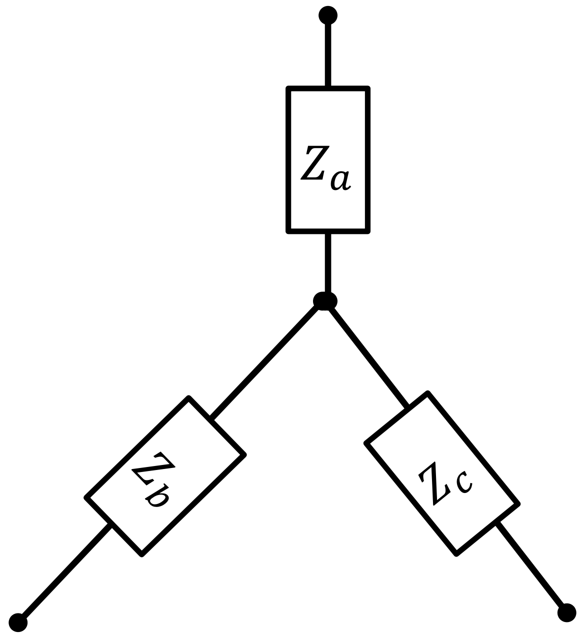
[存在しない]

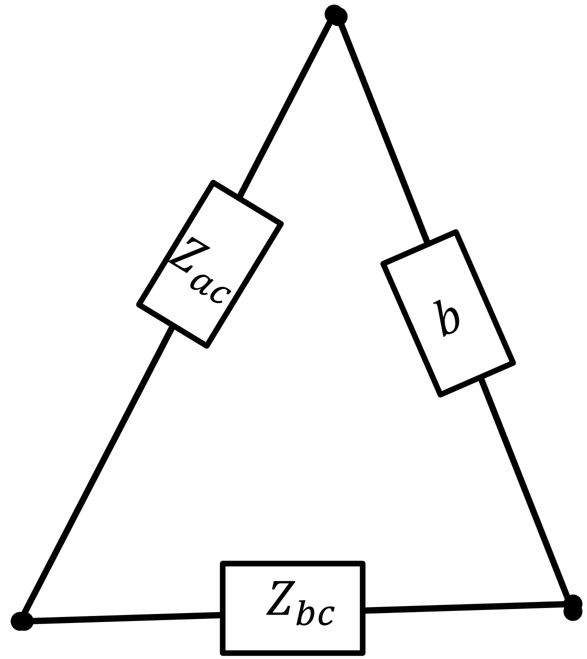
諸行列の応用例：　負荷のY−Δ変換

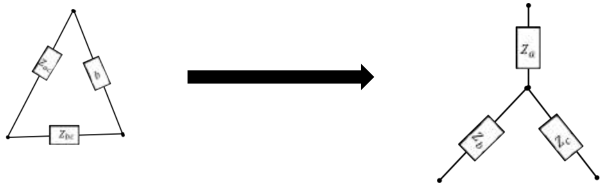




諸行列の応用例：負荷のY−Δ変換

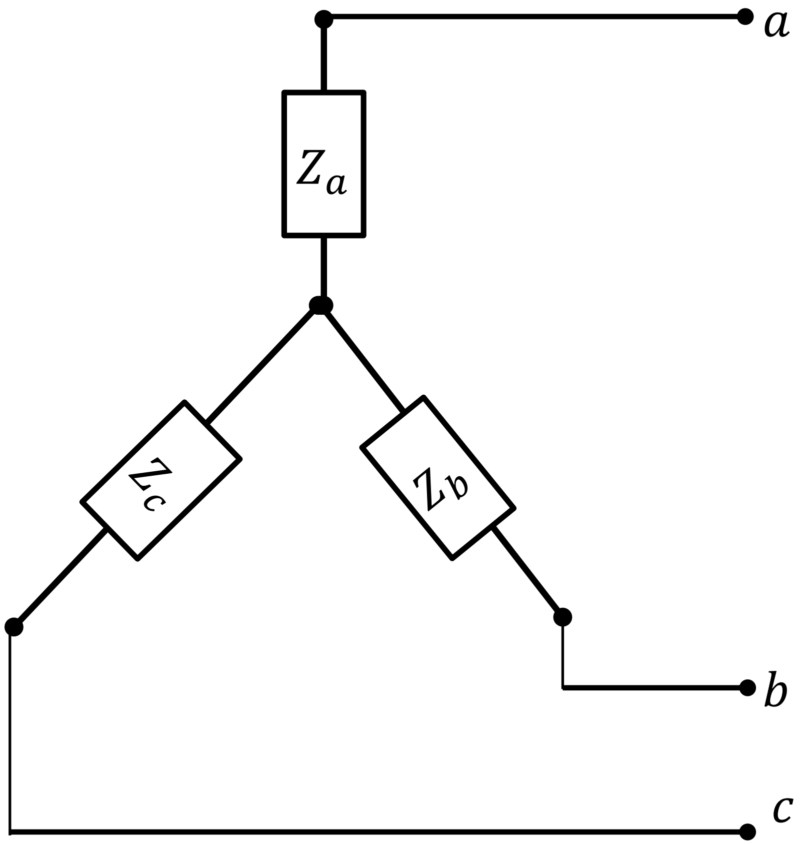




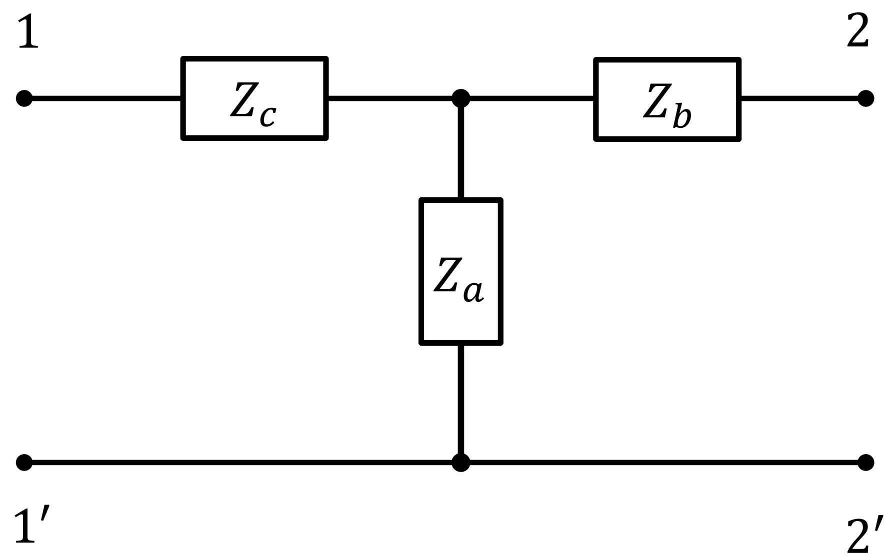


Zac
b
Zbc
Za
Zb
Zc

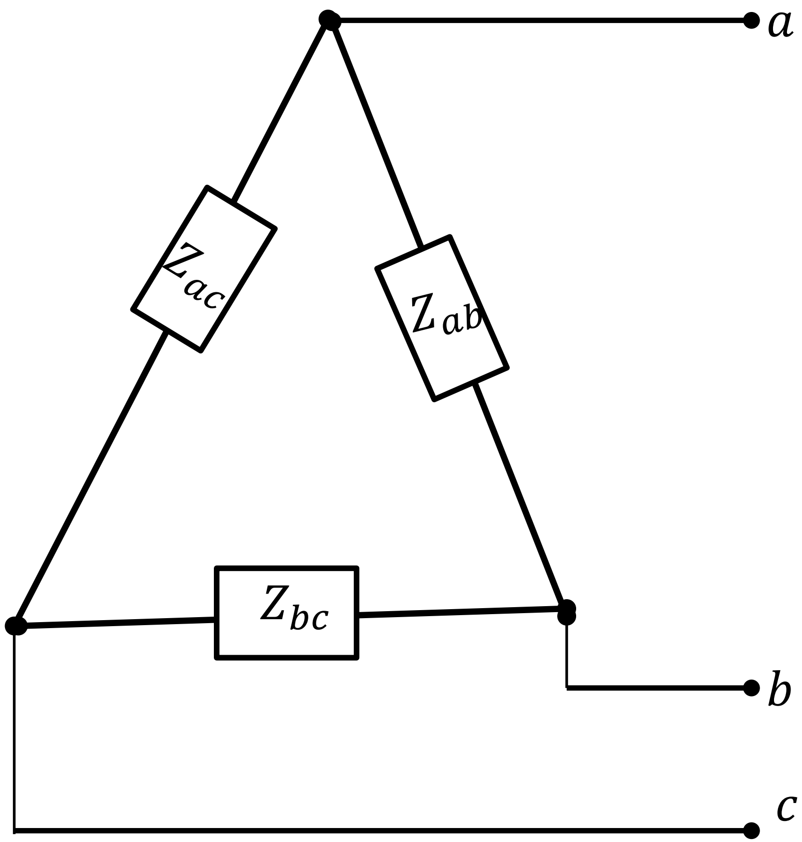
Y結線



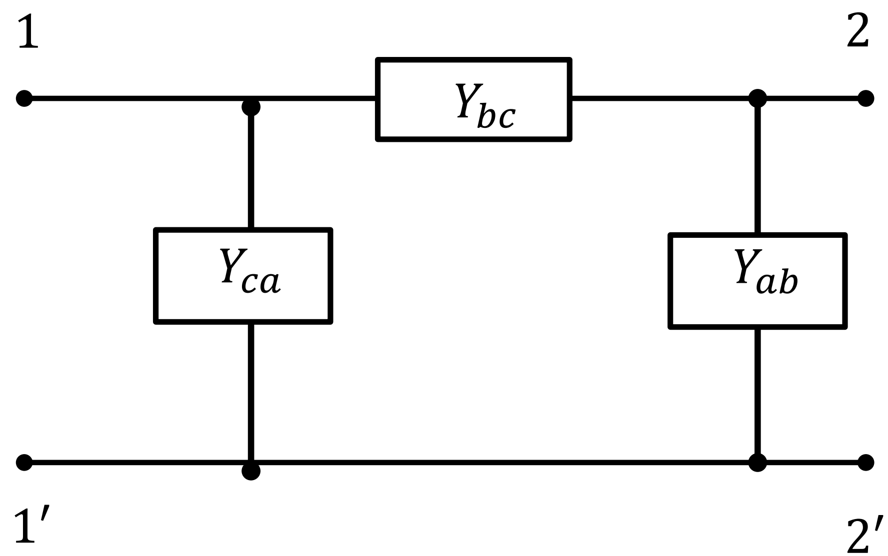
T型回路



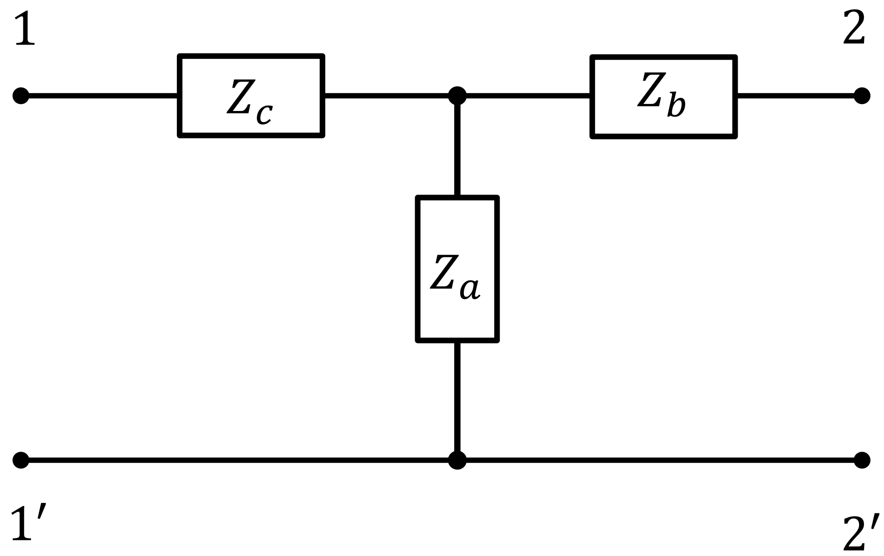
Δ結線



π型回路

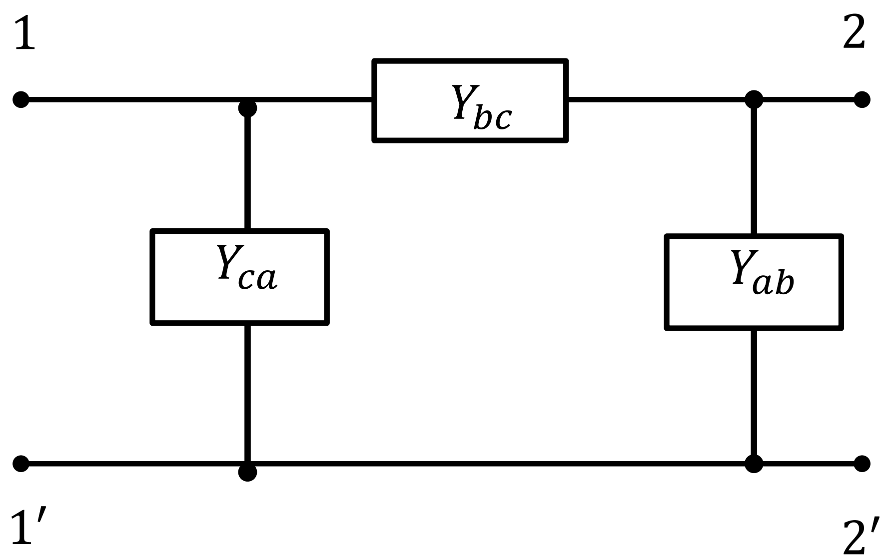


T型回路



$$[Z] = \begin{bmatrix} Z_c + Z_a & Z_a \\ Z_a & Z_a + Z_b \end{bmatrix}$$

π型回路



$$[Y] = \begin{bmatrix} Y_{ca} + Y_{bc} & -Y_{bc} \\ -Y_{bc} & Y_{bc} + Y_{ab} \end{bmatrix}$$

Z行列とY行列との逆行列　Δ→Y

$$[Y] = \begin{bmatrix} Y_{ca} + Y_{bc} & -Y_{bc} \\ -Y_{bc} & Y_{bc} + Y_{ab} \end{bmatrix}$$

$$\begin{bmatrix} Z_c + Z_a & Z_a \\ Z_a & Z_a + Z_b \end{bmatrix} = \frac{1}{\Delta_Y} \begin{bmatrix} Y_{22} & -Y_{12} \\ -Y_{21} & Y_{11} \end{bmatrix}$$

$$\begin{bmatrix} Z_c + Z_a & Z_a \\ Z_a & Z_a + Z_b \end{bmatrix} = \frac{1}{\Delta_Y} \begin{bmatrix} Y_{bc} + Y_{ab} & Y_{bc} \\ Y_{bc} & Y_{ca} + Y_{bc} \end{bmatrix}$$

$$\Delta = (Y_{bc} + Y_{ab})(Y_{ca} + Y_{bc}) - {Y_{bc}}^2$$

$$\Delta = Y_{bc}Y_{ab} + Y_{bc}Y_{ca} + Y_{ca}Y_{ab}$$

Δ→Y

$$\begin{bmatrix} Z_c + Z_a & Z_a \\ Z_a & Z_a + Z_b \end{bmatrix} = \frac{1}{\Delta_Y} \begin{bmatrix} Y_{bc} + Y_{ab} & Y_{bc} \\ Y_{bc} & Y_{ca} + Y_{bc} \end{bmatrix}$$

$$= \frac{1}{Y_{bc}Y_{ab} + Y_{bc}Y_{ca} + Y_{ca}Y_{ab}} \begin{bmatrix} Y_{bc} + Y_{ab} & Y_{bc} \\ Y_{bc} & Y_{ca} + Y_{bc} \end{bmatrix}$$

$$Z_a = \frac{Y_{bc}}{Y_{bc}Y_{ab} + Y_{bc}Y_{ca} + Y_{ca}Y_{ab}}$$

Δ→Y

$$Z_b = \frac{Y_{ca}}{Y_{bc}Y_{ab} + Y_{bc}Y_{ca} + Y_{ca}Y_{ab}}$$

$$Z_c = \frac{Y_{ab}}{Y_{bc}Y_{ab} + Y_{bc}Y_{ca} + Y_{ca}Y_{ab}}$$

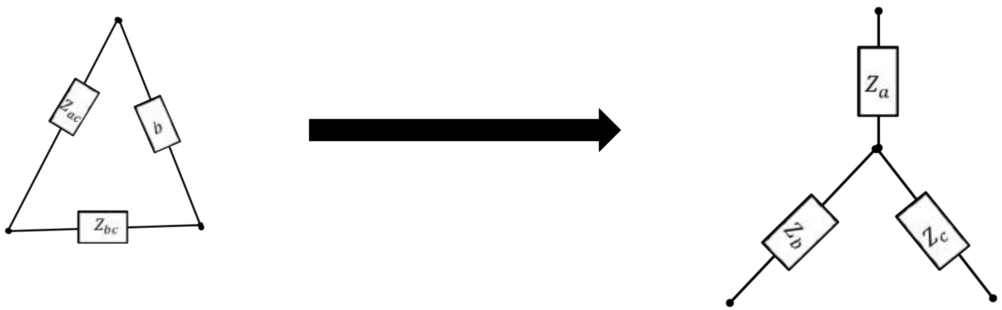
$$Y_{ab} = \frac{1}{Z_{ab}} \qquad Y_{bc} = \frac{1}{Z_{bc}} \qquad Y_{ca} = \frac{1}{Z_{ca}}$$

Δ→Y

$$Z_a = \frac{\frac{1}{Z_{bc}}}{\frac{1}{Z_{bc}}\frac{1}{Z_{ab}} + \frac{1}{Z_{bc}}\frac{1}{Z_{ca}} + \frac{1}{Z_{ca}}\frac{1}{Z_{ab}}}$$

$$Z_a = \frac{Z_{ca}Z_{ab}}{Z_{ca} + Z_{ab} + Z_{bc}} \qquad Z_b = \frac{Z_{ab}Z_{bc}}{Z_{ca} + Z_{ab} + Z_{bc}}$$

$$Z_c = \frac{Z_{bc}Z_{ca}}{Z_{ca} + Z_{ab} + Z_{bc}}$$



$$[Z] = \begin{bmatrix} Z_c + Z_a & Z_a \\ Z_a & Z_a + Z_b \end{bmatrix}$$

$$\begin{bmatrix} Y_{ca} + Y_{bc} & -Y_{bc} \\ -Y_{bc} & Y_{bc} + Y_{ab} \end{bmatrix} = \frac{1}{\Delta_Z} \begin{bmatrix} Z_{22} & -Z_{12} \\ -Z_{21} & Z_{11} \end{bmatrix}$$

$$\begin{bmatrix} Y_{ca} + Y_{bc} & -Y_{bc} \\ -Y_{bc} & Y_{bc} + Y_{ab} \end{bmatrix} = \frac{1}{\Delta_Z} \begin{bmatrix} Z_a + Z_b & -Z_a \\ -Z_a & Z_c + Z_a \end{bmatrix}$$

$$\Delta = (Z_c + Z_a)(Z_a + Z_b) - {Z_a}^2$$

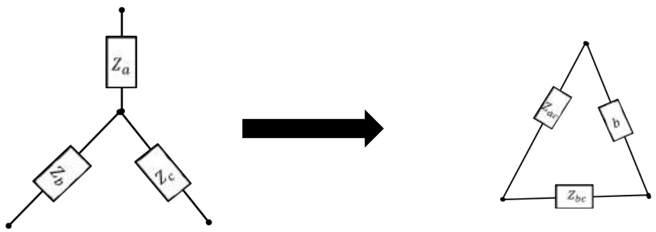
$$\Delta = Z_a Z_b + Z_b Z_c + Z_c Z_a$$

$$\begin{bmatrix} Y_{ca}+Y_{bc} & -Y_{bc} \\ -Y_{bc} & Y_{bc}+Y_{ab} \end{bmatrix} = \frac{1}{\Delta_Z} \begin{bmatrix} Z_a+Z_b & -Z_a \\ -Z_a & Z_c+Z_a \end{bmatrix}$$

$$= \frac{1}{Z_aZ_b + Z_bZ_c + Z_cZ_a} \begin{bmatrix} Z_a+Z_b & -Z_a \\ -Z_a & Z_c+Z_a \end{bmatrix}$$

$$Y_{bc} = \frac{Z_a}{Z_aZ_b + Z_bZ_c + Z_cZ_a} \qquad Y_{ca} = \frac{Z_b}{Z_aZ_b + Z_bZ_c + Z_cZ_a}$$

$$Y_{ab} = \frac{Z_c}{Z_aZ_b + Z_bZ_c + Z_cZ_a}$$



$$Y_{ab} = \frac{1}{Z_{ab}} \qquad Y_{bc} = \frac{1}{Z_{bc}} \qquad Y_{ca} = \frac{1}{Z_{ca}}$$

Y→Δ

$$Z_{bc} = \frac{Z_a Z_b + Z_b Z_c + Z_c Z_a}{Z_a} \qquad Z_{ab} = \frac{Z_a Z_b + Z_b Z_c + Z_c Z_a}{Z_c}$$

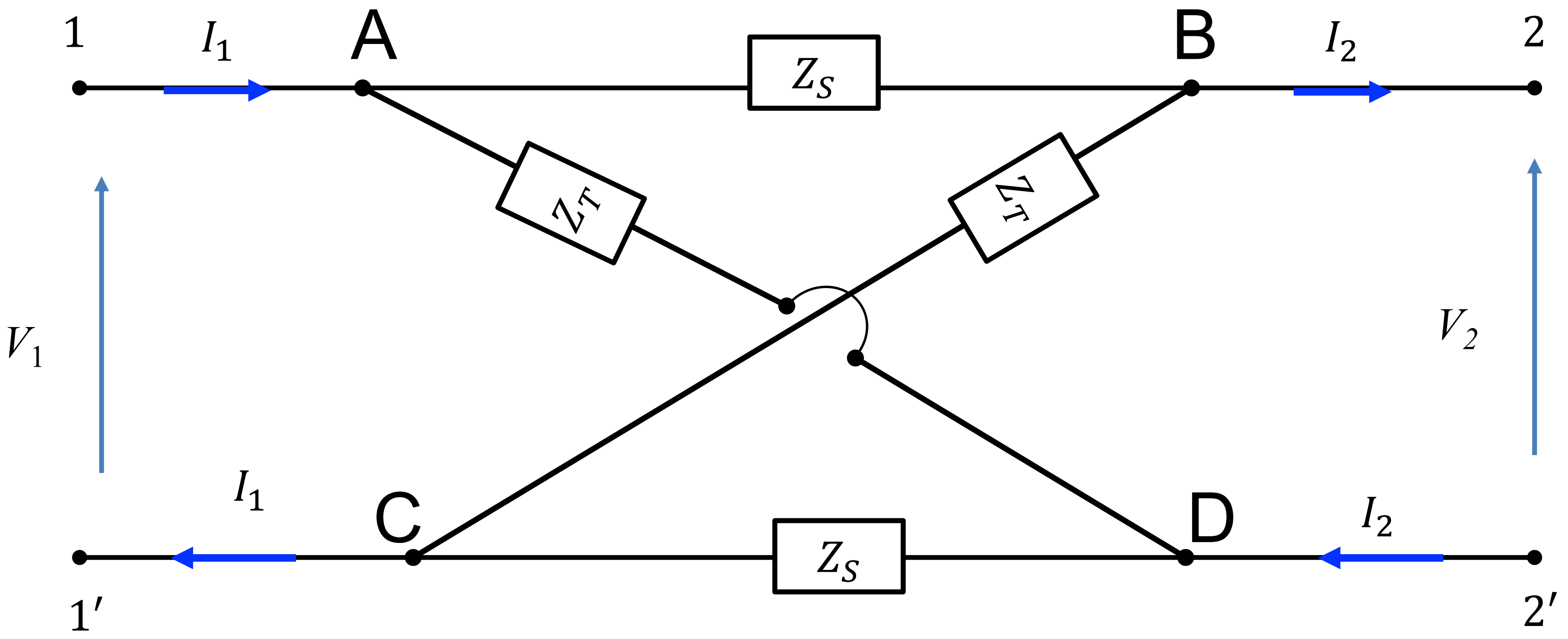
$$Z_{ac} = \frac{Z_a Z_b + Z_b Z_c + Z_c Z_a}{Z_b}$$

Δ → Y

$$Z_a = \frac{Z_{ca} Z_{ab}}{Z_{ca} + Z_{ab} + Z_{bc}} \qquad Z_b = \frac{Z_{ab} Z_{bc}}{Z_{ca} + Z_{ab} + Z_{bc}}$$

$$Z_c = \frac{Z_{bc} Z_{ca}}{Z_{ca} + Z_{ab} + Z_{bc}}$$

対称格子型回路： F行列を求める



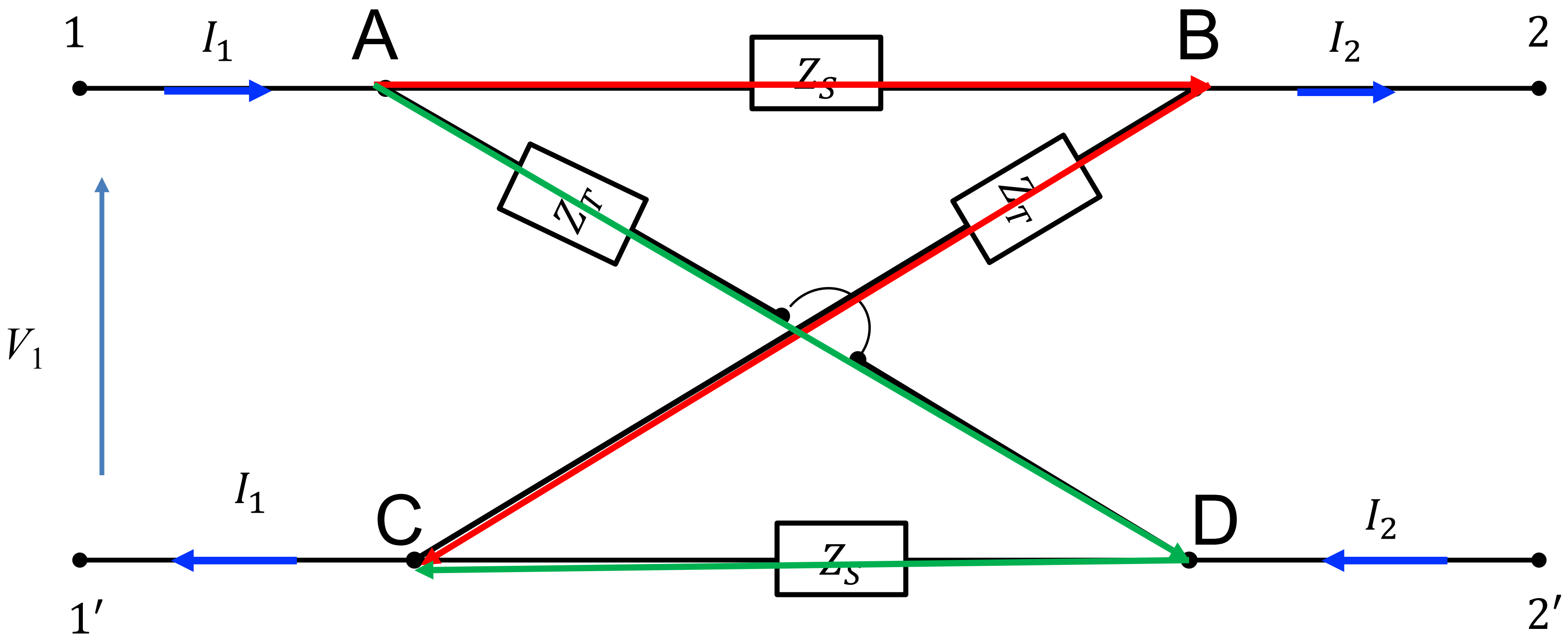
$$\dot{A} = \left.\frac{\dot{V}_1}{\dot{V}_2}\right|_{\dot{I}_2=0} = $$ 出力端開放　電圧転送比

$$\dot{B} = \left.\frac{\dot{V}_1}{\dot{I}_2}\right|_{\dot{V}_2=0} = $$ 出力端短絡　伝達インピーダンス

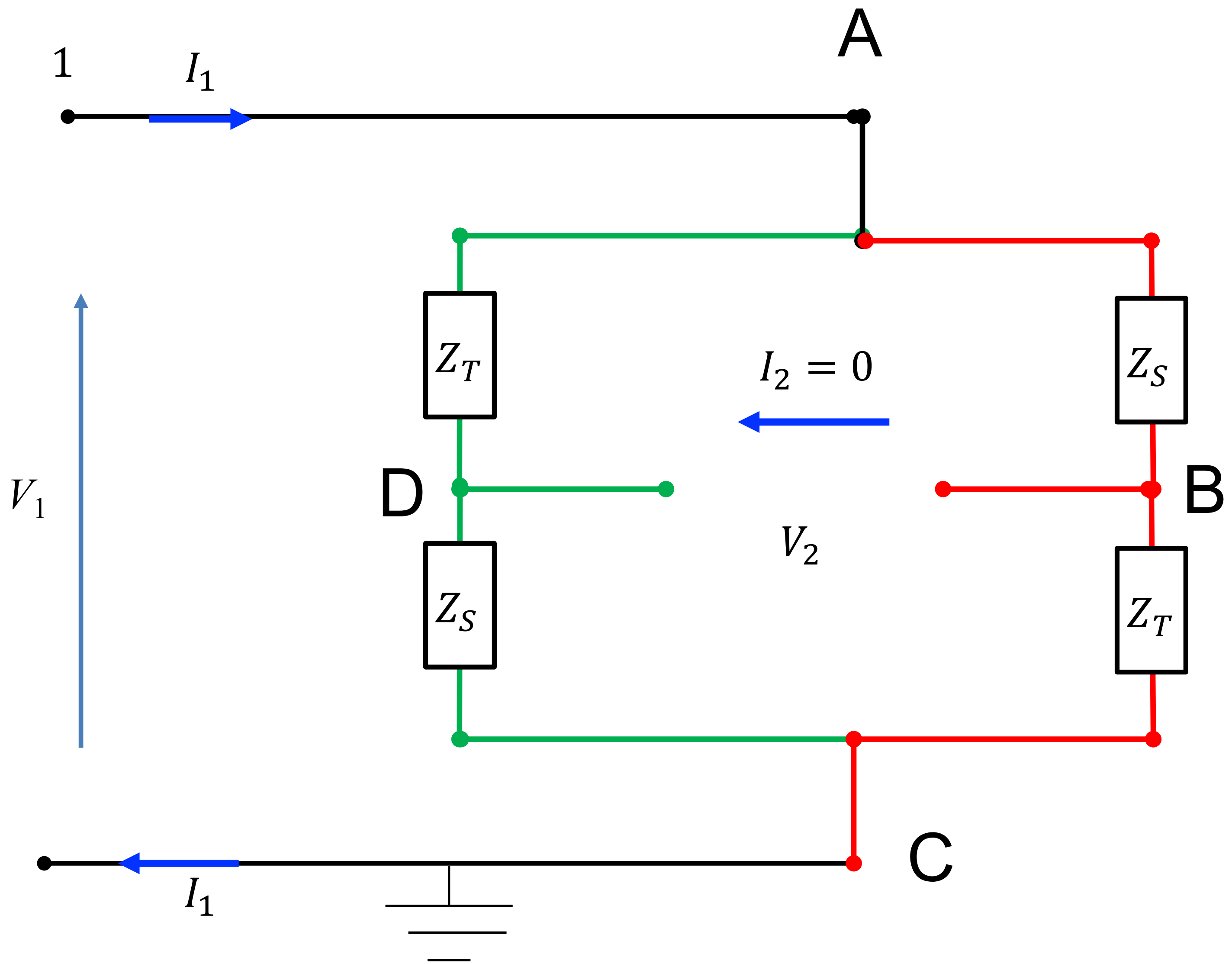
$$\dot{C} = \left.\frac{\dot{I}_1}{\dot{V}_2}\right|_{\dot{I}_2=0} = $$ 出力端開放　伝達アドミタンス

$$\dot{D} = \left.\frac{\dot{I}_1}{\dot{I}_2}\right|_{\dot{V}_2=0} = $$ 出力端短絡　電流転送比

対称格子型回路



$I_2 = 0$　$2-2'$を開放した場合



$$\dot{A} = \left.\frac{\dot{V}_1}{\dot{V}_2}\right|_{\dot{I}_2=0}$$

$$V_B = \frac{Z_T}{Z_S + Z_T} V_1$$

$$V_D = \frac{Z_S}{Z_S + Z_T} V_1$$

$$V_2 = V_B - V_D = \frac{Z_T - Z_S}{Z_S + Z_T} V_1$$

$$A = V_B - V_D = \frac{V_1}{V_2} = \frac{Z_T + Z_S}{Z_S - Z_T}$$

$$\dot{C} = \left.\frac{\dot{I}_1}{\dot{V}_2}\right|_{\dot{I}_2=0}$$

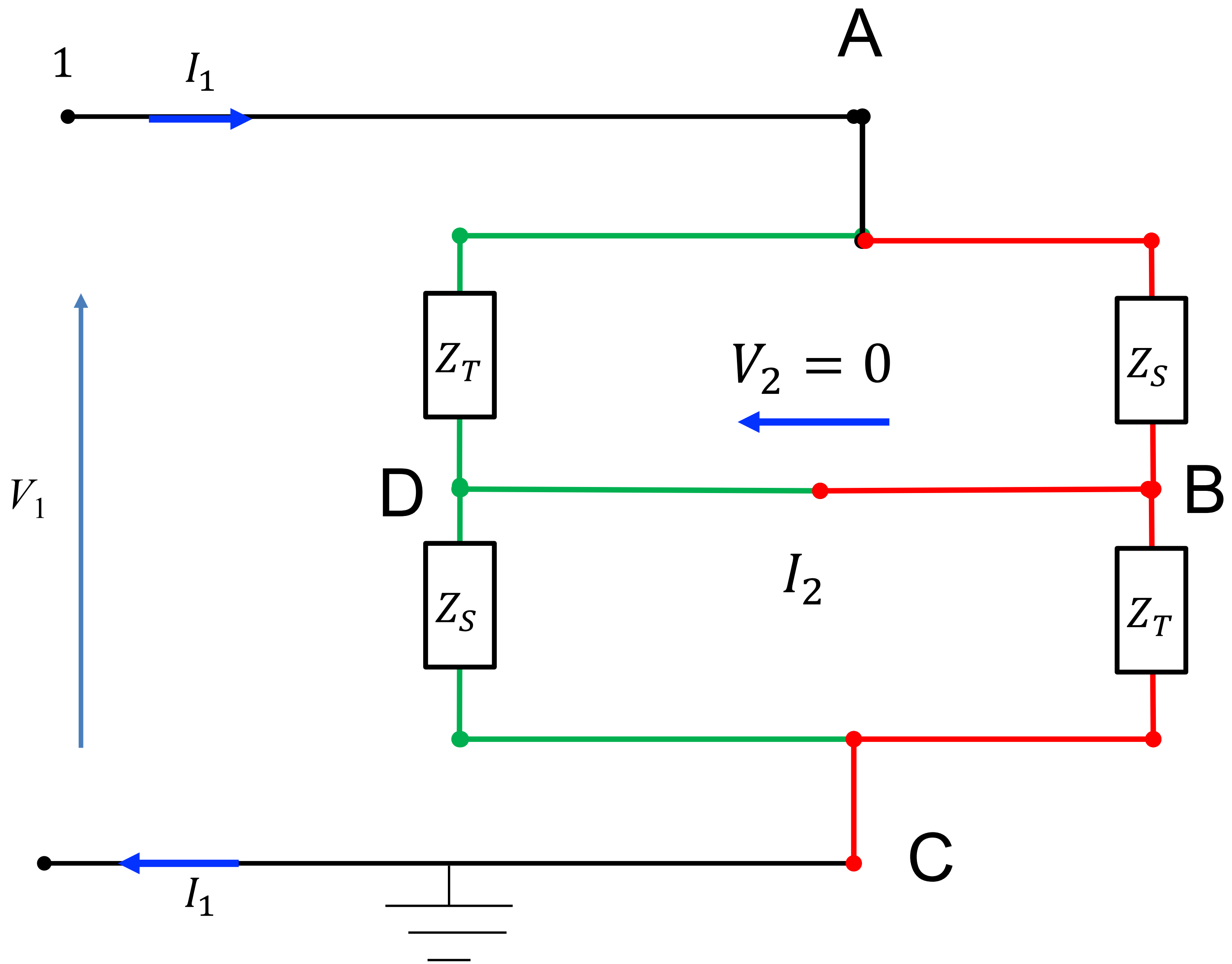
$$Z_{AC} = \frac{Z_S + Z_T}{2}$$

$$I_1 = \frac{V_1}{Z_{AC}} = \frac{2V_1}{Z_S + Z_T}$$

$$V_2 = \frac{Z_T - Z_S}{Z_S + Z_T} V_1$$

$$C = \frac{I_1}{V_2} = \frac{2}{Z_T - Z_S}$$

$V_2 = 0$　$2 - 2'$を短絡した場合



$$\dot{B} = \left.\frac{\dot{V}_1}{\dot{I}_2}\right|_{\dot{V}_2=0}$$

$$Z_{AC} = 2\frac{Z_S Z_T}{Z_S + Z_T}$$

$$I_1 = \frac{V_1}{Z_{AC}} = \frac{Z_S + Z_T}{2Z_S Z_T}V_1$$

$$I_A = \frac{Z_T}{Z_S + Z_T}I_1$$

$$I_C = \frac{Z_s}{Z_S + Z_T}I_1$$

$$I_2 = I_A - I_C = \frac{Z_T - Z_s}{Z_S + Z_T}I_1 = \frac{Z_T - Z_s}{Z_S + Z_T}\frac{Z_S + Z_T}{2Z_S Z_T}V_1$$

$$\dot{D} = \left.\frac{\dot{I}_1}{\dot{I}_2}\right|_{\dot{V}_2=0}$$

$$I_2 = \frac{Z_T - Z_s}{2Z_S Z_T} V_1$$

$$\boldsymbol{B} = \frac{V_1}{I_2} = \frac{2Z_S Z_T}{Z_T - Z_s}$$

$$\boldsymbol{D} = \frac{\boldsymbol{I}_1}{\boldsymbol{I}_2} = \frac{(Z_S + Z_T)V_1}{2Z_S Z_T} \frac{2Z_S Z_T}{(Z_T - Z_s)V_1} = \frac{Z_S + Z_T}{Z_T - Z_s}$$

$$A = D = \frac{Z_T + Z_S}{Z_S - Z_T} \qquad B = \frac{2Z_S Z_T}{Z_T - Z_s} \qquad C = \frac{2}{Z_T - Z_S}$$

$$\boldsymbol{F} = \begin{bmatrix} \dfrac{Z_T + Z_S}{Z_S - Z_T} & \dfrac{2Z_S Z_T}{Z_T - Z_s} \\ \dfrac{2}{Z_T - Z_S} & \dfrac{Z_T + Z_S}{Z_S - Z_T} \end{bmatrix}$$

対称格子型回路におけるZ行列とF行列の変換

$$\boldsymbol{F} = \begin{bmatrix} \dfrac{\boldsymbol{Z_T + Z_S}}{\boldsymbol{Z_S - Z_T}} & \dfrac{\boldsymbol{2Z_S Z_T}}{\boldsymbol{Z_T - Z_s}} \\ \dfrac{\boldsymbol{2}}{\boldsymbol{Z_T - Z_S}} & \dfrac{\boldsymbol{Z_T + Z_S}}{\boldsymbol{Z_S - Z_T}} \end{bmatrix}$$

$$[Z] = \begin{bmatrix} A/C & 1/C \\ 1/C & D/C \end{bmatrix}$$

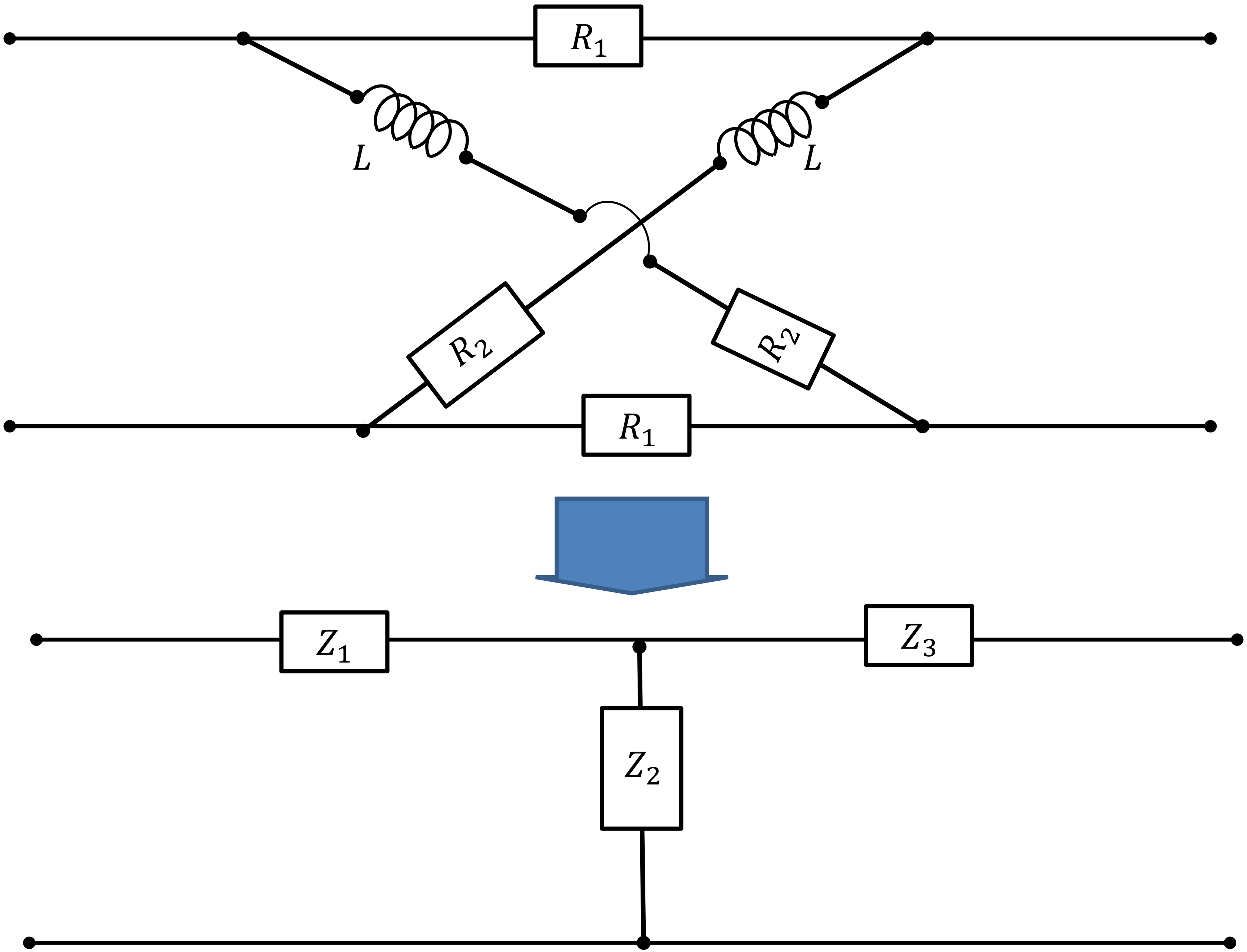
$$A/C = \frac{\mathbf{Z}_T + \mathbf{Z}_S}{\mathbf{Z}_S - \mathbf{Z}_T} * \frac{\mathbf{Z}_T - \mathbf{Z}_S}{\mathbf{2}} = \frac{\mathbf{Z}_T + \mathbf{Z}_S}{\mathbf{2}}$$

$$1/C = \frac{\mathbf{Z}_T - \mathbf{Z}_S}{\mathbf{2}} \qquad D/C = \frac{\mathbf{Z}_T + \mathbf{Z}_S}{\mathbf{2}}$$

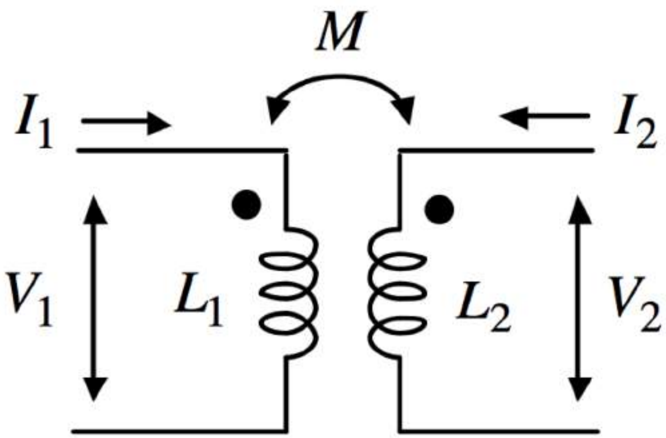
$$[Z] = \frac{\mathbf{1}}{\mathbf{2}}\begin{bmatrix} \mathbf{Z}_T + \mathbf{Z}_S & \mathbf{Z}_T - \mathbf{Z}_S \\ \mathbf{Z}_T - \mathbf{Z}_S & \mathbf{Z}_T + \mathbf{Z}_S \end{bmatrix}$$

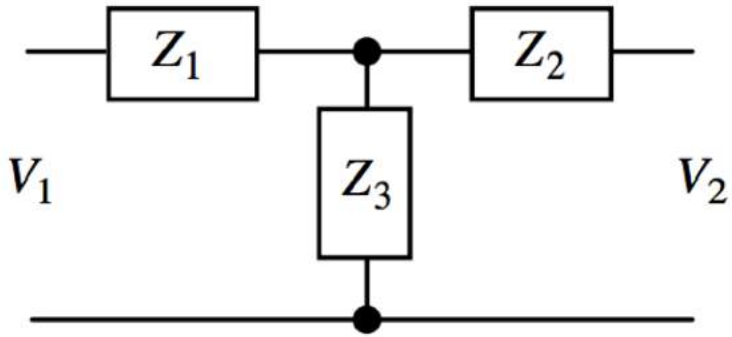
(a)

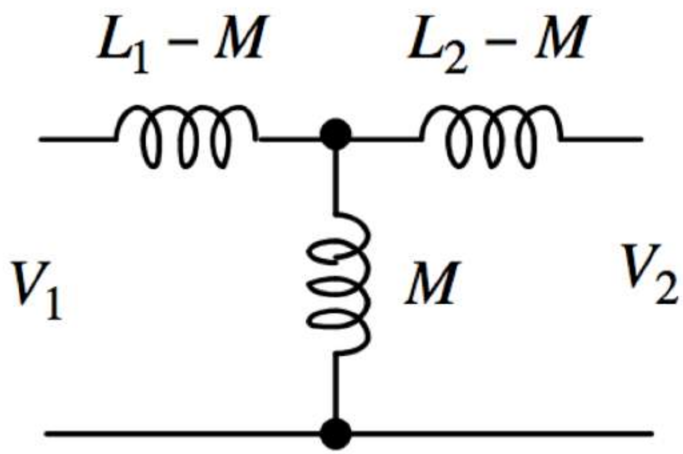
(b)

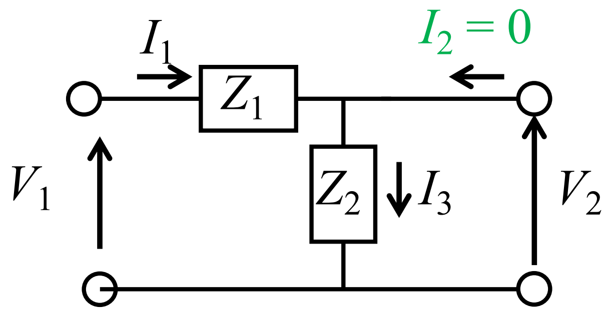


図に示した対称格子形回路をこれと等価な図(b)のT型回路に等価変換したい、インピーダンスZ_1、Z_2、Z_3,を決定せよ。ただし、$R_1 = 5[\Omega]$、$R_2 = 15[\Omega]$、$L = 10[mH]$、角周波数$w = 1000[rad/s]$

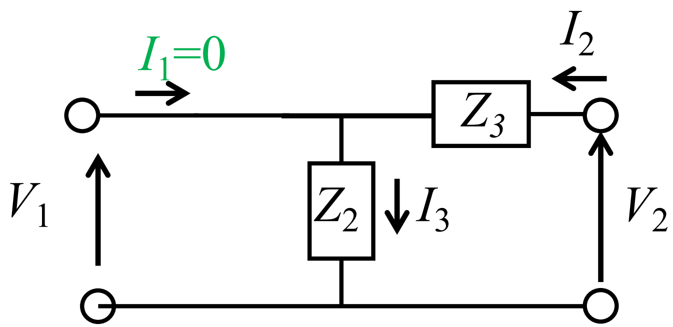

M
I1
I2
V1
L1
L2
V2


Z1
Z2
Z3
V1
V2


L1 – M
L2 – M
V1
M
V2



I_1
$I_2 = 0$
Z_1
V_1
Z_2
I_3
V_2



I_1=0
I_2
Z_3
V_1
Z_2
I_3
V_2