i-PERC
電気通信大学

基礎電気回路CH-11

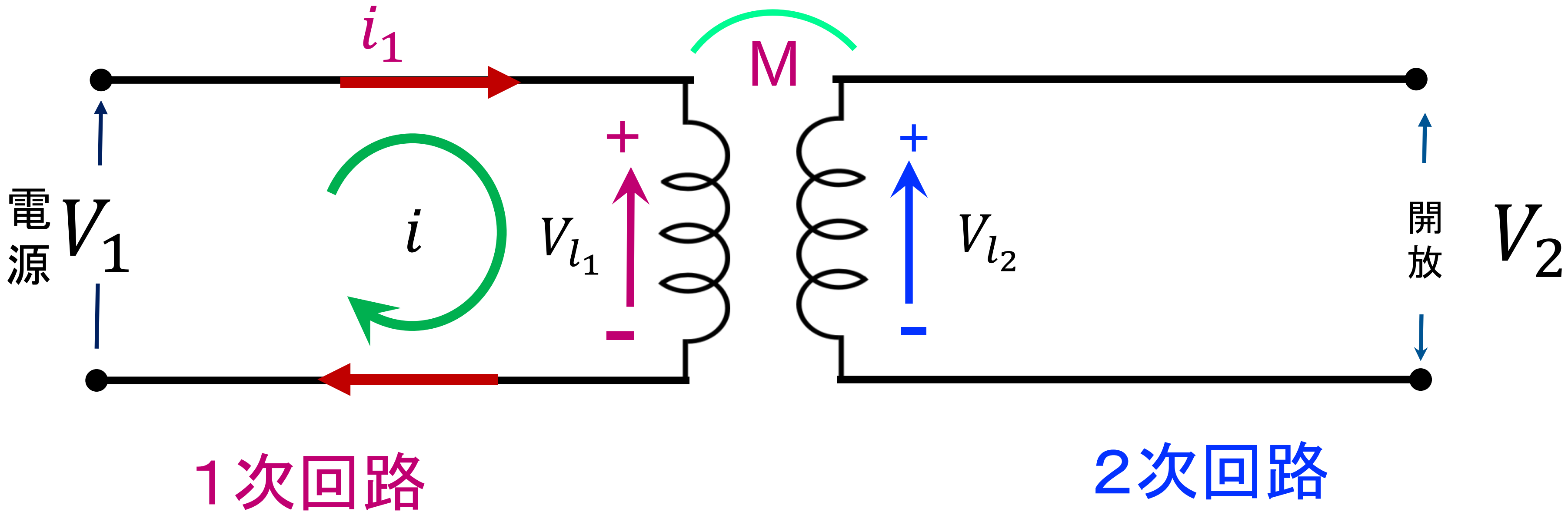
相互誘導回路

等価回路

変圧器回路

理想変圧器

相互誘導インダクタンス回路にKVL法則を応用する



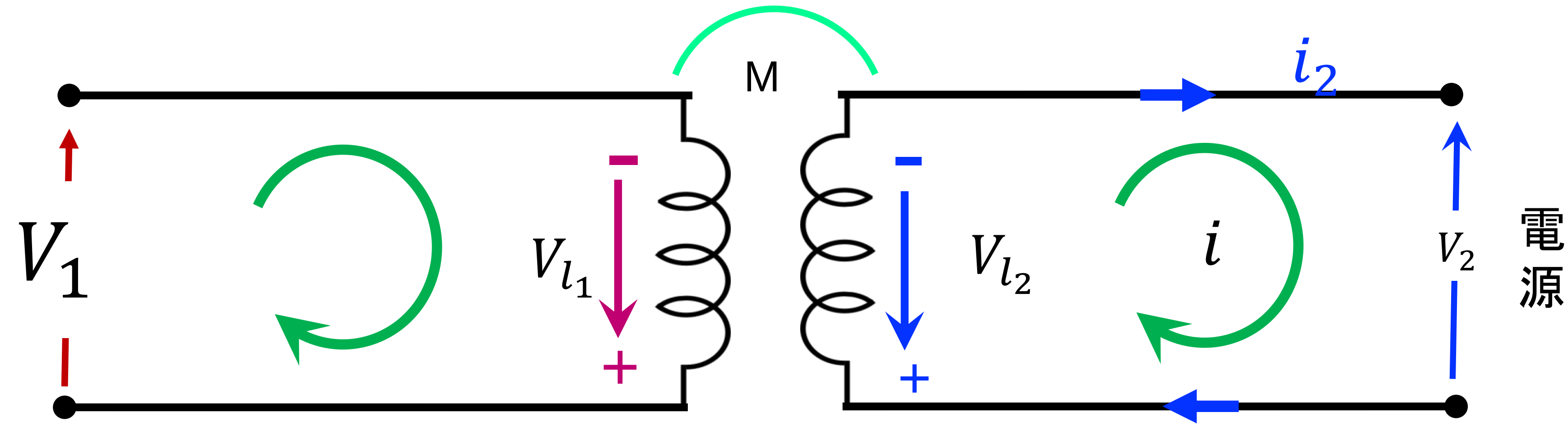
$$V_{l_1} - V_1 = 0$$

$$V_2 - V_{l_2} = 0$$

$$V_1 = V_{l_1} = L_1 \frac{di_1}{dt}$$

$$V_2 = V_{l_2} = \boldsymbol{M_{21}} \frac{di_1}{dt}$$

相互誘導インダクタンス回路にKVL法則を応用する



2次回路

$$-V_{l_1} + V_1 = 0$$

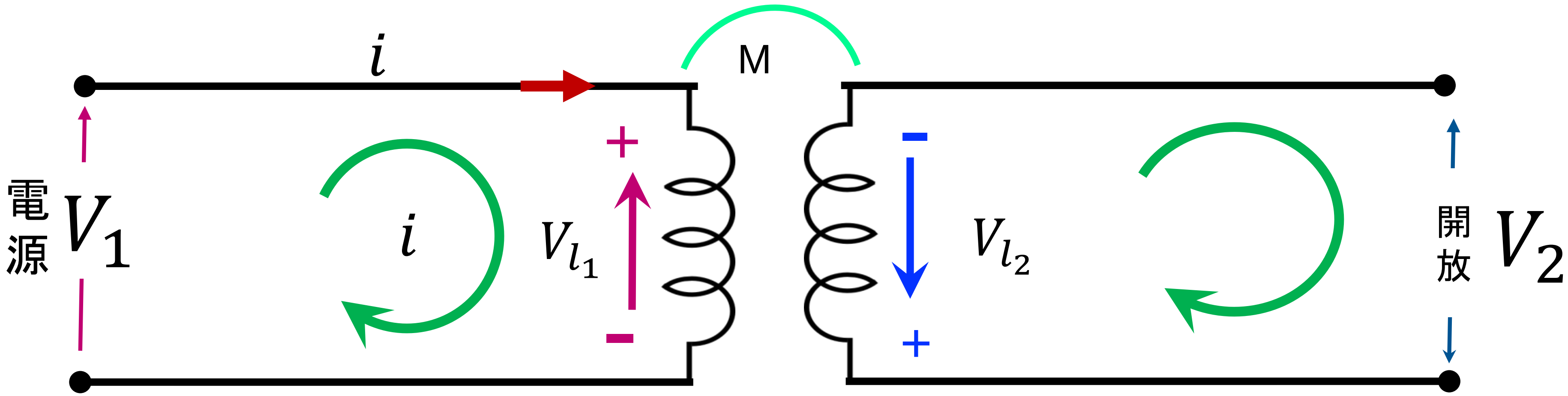
$$V_1 = V_{l_1} = \boldsymbol{M}_{12} \frac{d\boldsymbol{i}_2}{dt}$$

1次回路

$$-V_2 + V_{l_2} = 0$$

$$V_2 = V_{l_2} = L_2 \frac{di_2}{dt}$$

相互誘導インダクタンス回路にKVL法則を応用する:逆巻き



1次回路

$$V_{l_1} - V_1 = 0$$

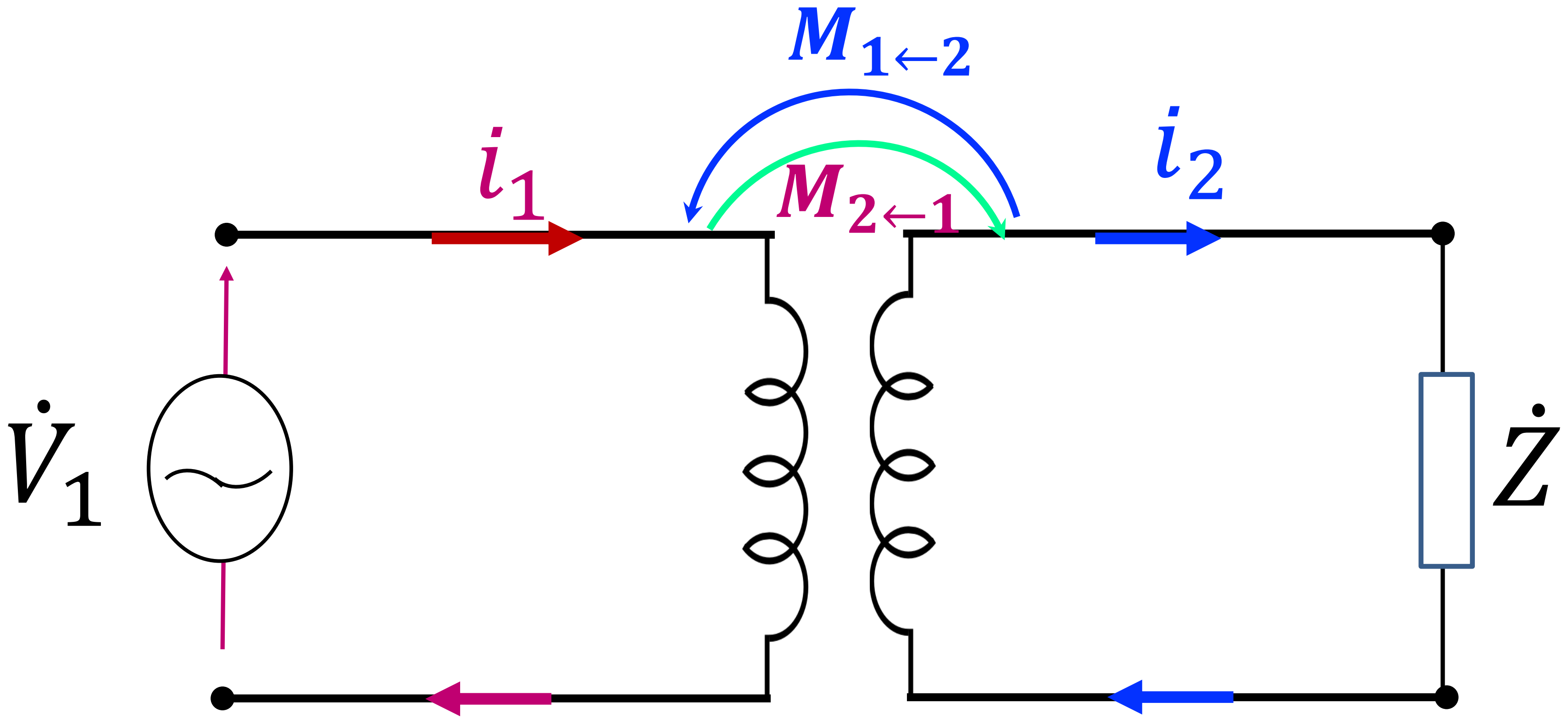
$$V_1 = V_{l_1} = L_1 \frac{di_1}{dt}$$

2次回路

$$V_2 + V_{l_2} = 0$$

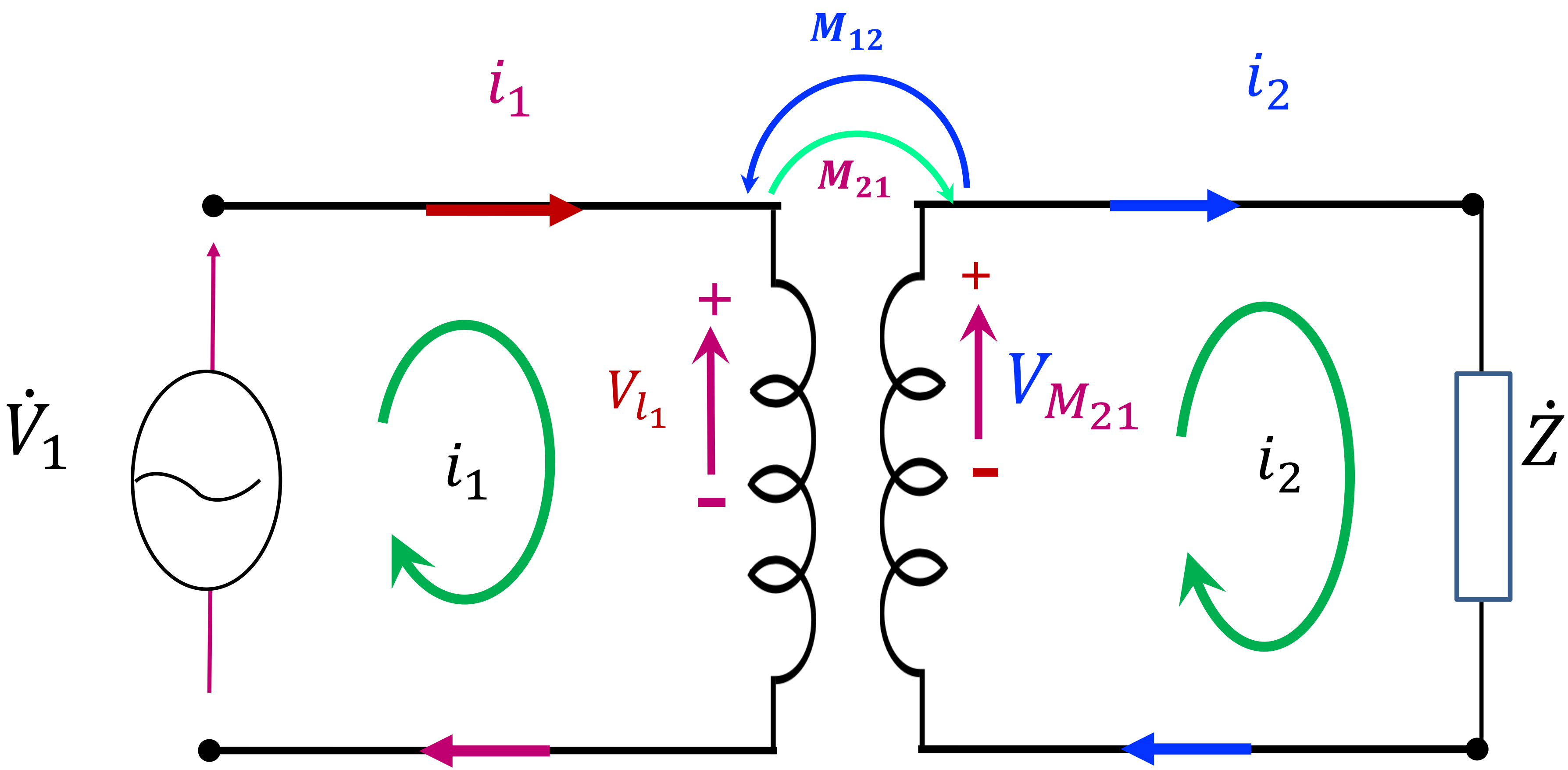
$$V_2 = -V_{l_2} = M_{21} \frac{di_1}{dt}$$

電磁誘導結合回路の拡張：交流電源と負荷を入れる



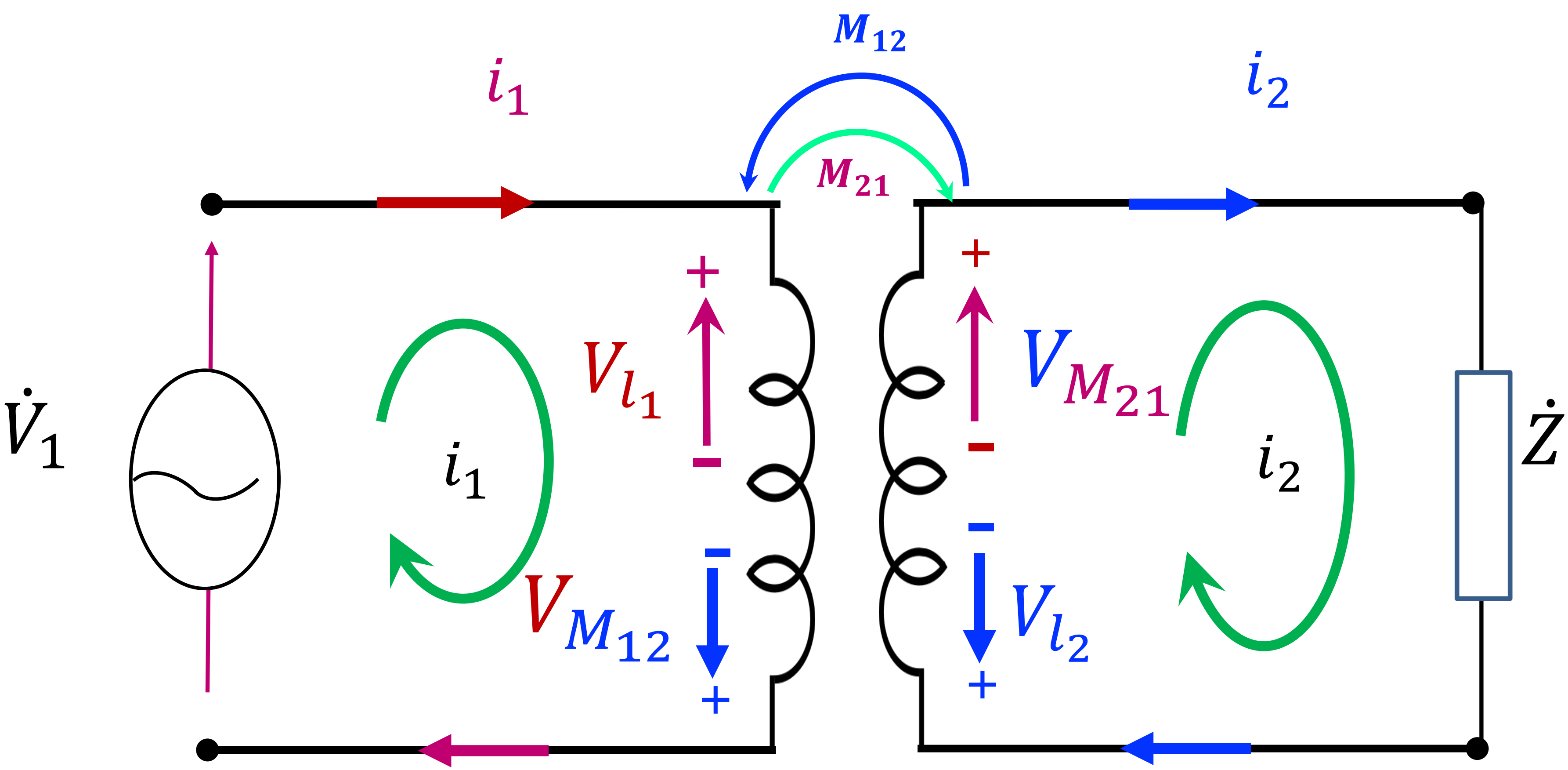
電磁誘導結合回路:KVL法則

$$\boldsymbol{M} = \boldsymbol{M_{12}} = \boldsymbol{M_{21}}$$

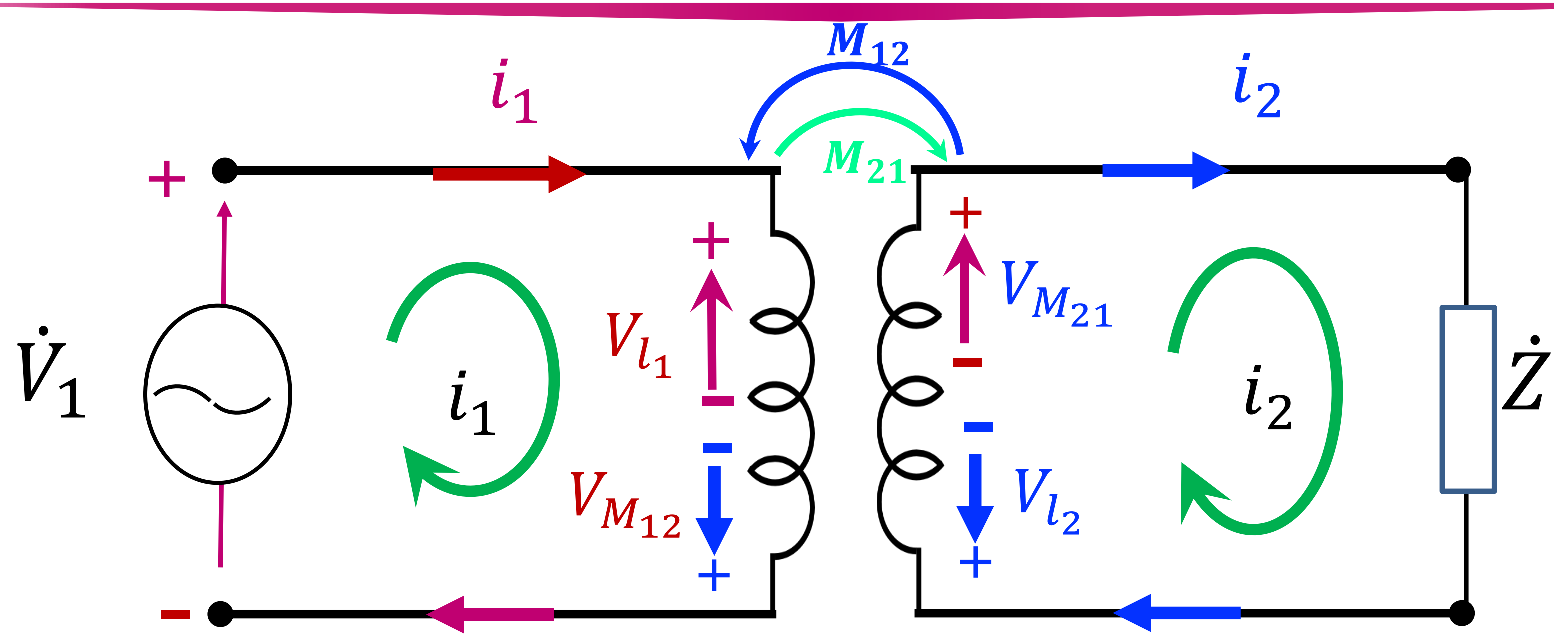


電磁誘導結合回路:KVL法則

$$M = M_{12} = M_{21}$$



電磁誘導結合回路：ループ1におけるKVL法則



(a) $V_{l_1} + V_{M_{12}} - \dot{V}_1 = 0$

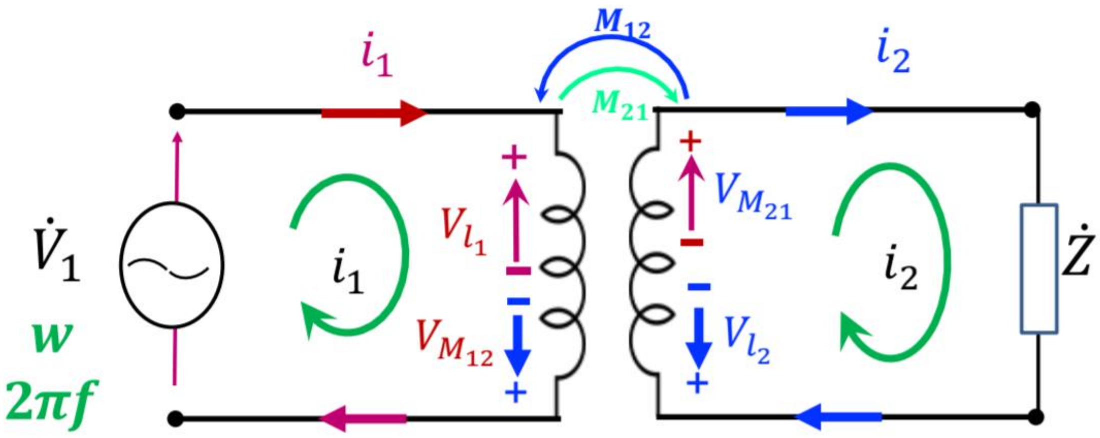
(c) $V_{l_1} - V_{M_{12}} - \dot{V}_1 = 0$

(b) $V_{l_1} + V_{M_{12}} + \dot{V}_1 = 0$

(d) $V_{l_1} - V_{M_{12}} + \dot{V}_1 = 0$

正解は:

電磁誘導結合回路:KVL法則

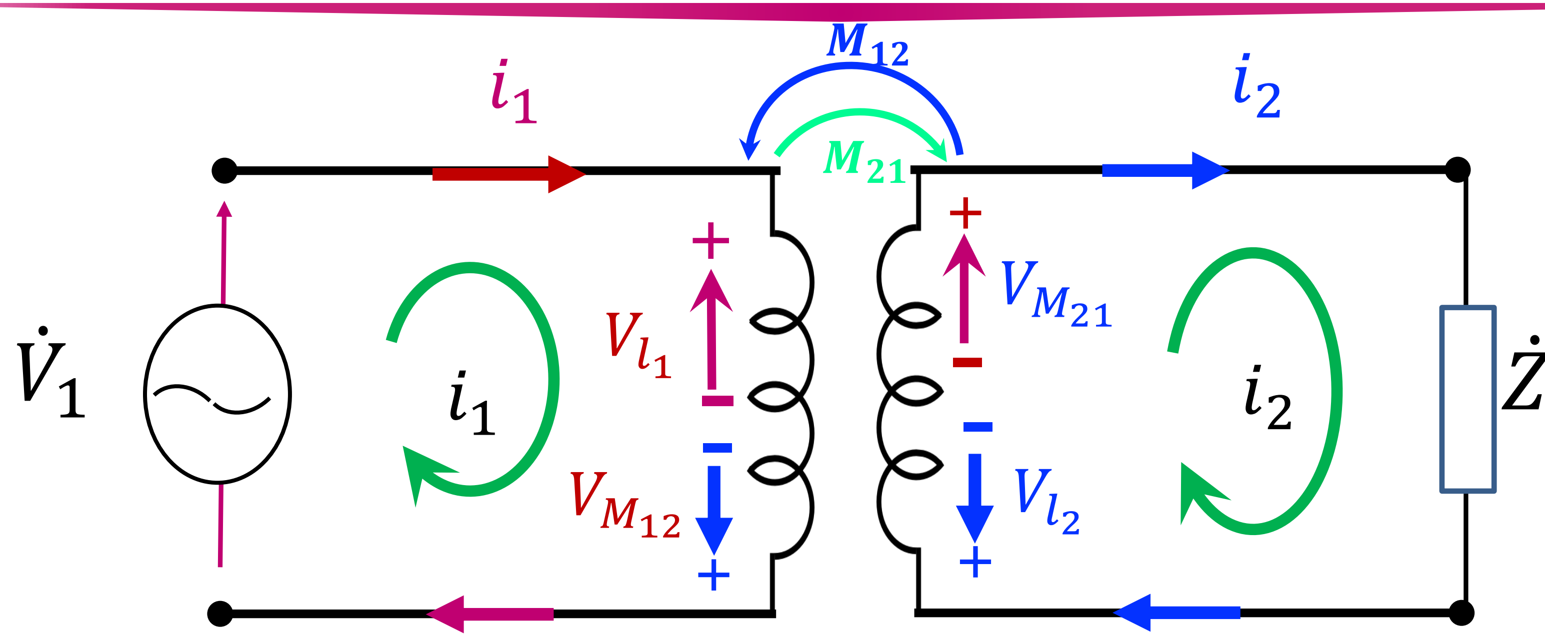


$$V_{l_1} - V_{M_{12}} - \dot{V}_1 = 0$$

$$L_1 \frac{di_1}{dt} - M_{12}\frac{di_2}{dt} - \dot{V}_1 = 0$$

$$jwL_1 i_1 - jwM_{12}\ i_2 = \dot{V}_1$$

電磁誘導結合回路:KVL法則



ループ2におけるKVL法則:

(a) $i_2\dot{Z} + V_{M_{21}} - V_{l_2} = 0$

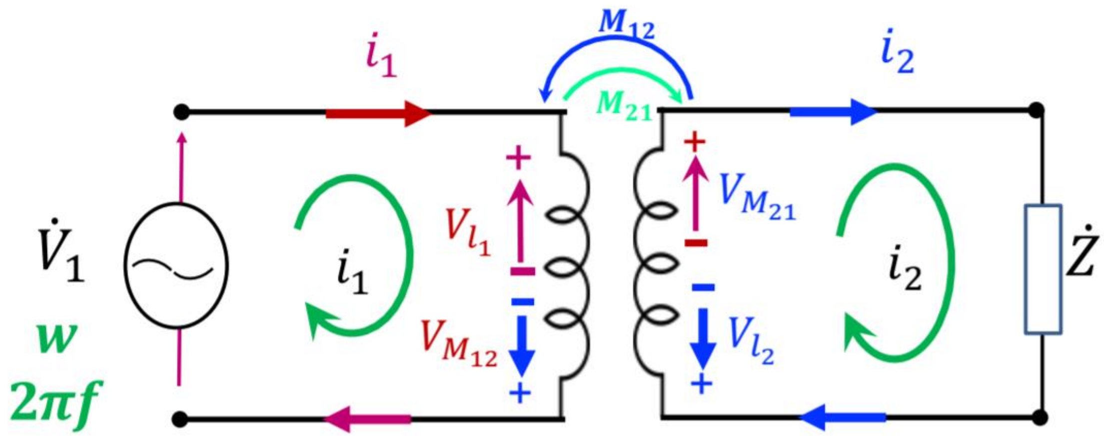
(c) $V_{M_{21}} - V_{l_2} + i_2\dot{Z} = 0$

(b) $-V_{M_{21}} + V_{l_2} + i_2\dot{Z} = 0$

(d) $V_{l_2} + V_{M_{21}} - i_2\dot{Z} = 0$

正解は:

電磁誘導結合回路:KVL法則

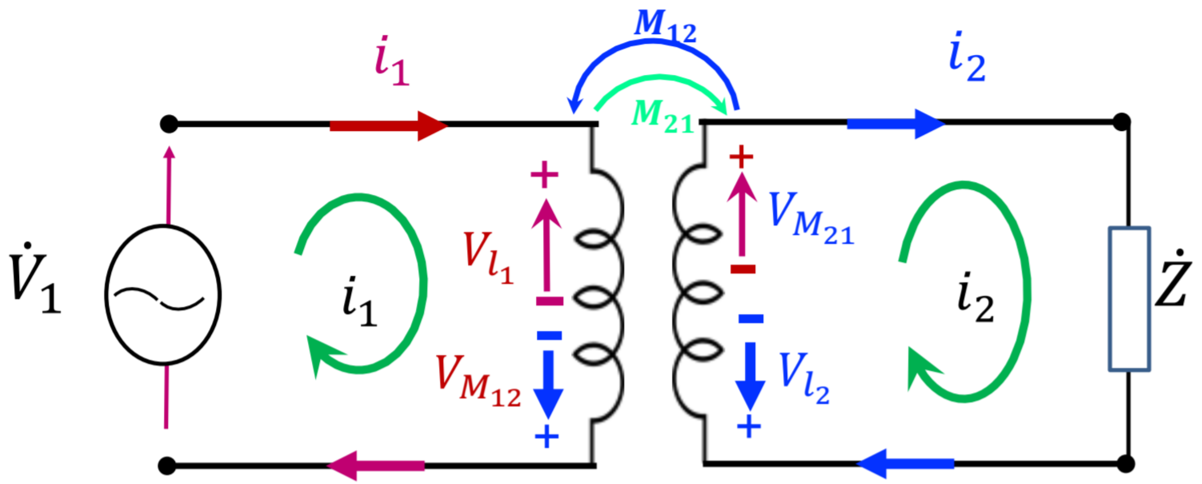


$$-V_{M_{21}} + V_{l_2} + i_2\dot{Z} = 0$$

$$M_{21}\frac{di_1}{dt} - L_2\frac{di_2}{dt} = \dot{Z}i_2$$

$$jwM_{12}\, i_1 - jwL_2\, i_2 = \dot{Z}i_2$$

電磁誘導結合回路：KVL法則



1次回路

$$\dot{V}_1 = jwL_1 i_1 - jwM_{12}\, i_2$$

2次回路

$$\dot{Z} i_2 = jwM_{21}\, i_1 - jwL_2\, i_2$$

電磁誘導結合回路：1次回路におけるインピーダンス

$$\boldsymbol{M} = \boldsymbol{M_{12}} = \boldsymbol{M_{21}}$$

$$\dot{Z} i_2 = jwM_{21}\ i_1 - \ jwL_2\ i_2$$

$$i_2 = \frac{jwM}{jwL_2 + \dot{Z}}\, i_1$$

$$\dot{V}_1 = jwL_1 i_1 - jwM_{12}\ i_2$$

$$\dot{V}_1 = jwL_1 i_1 - jwM \frac{jwM}{jwL_2 + \dot{Z}}\ i_1$$

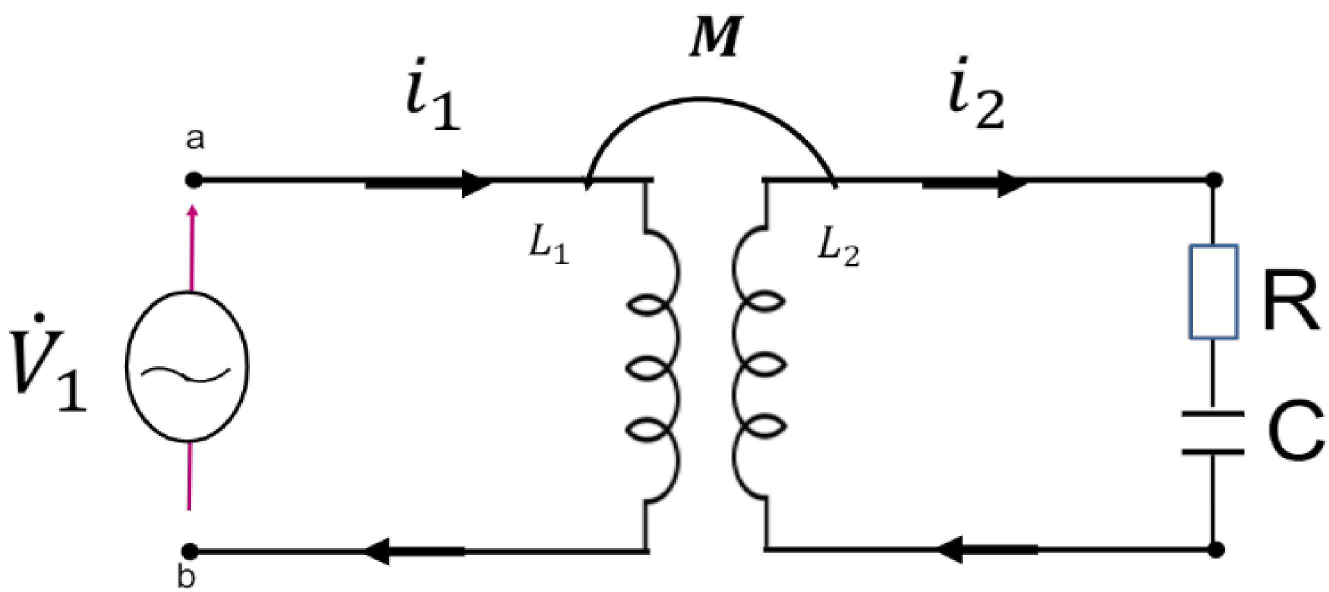
電磁誘導結合回路：1次回路におけるインピーダンス

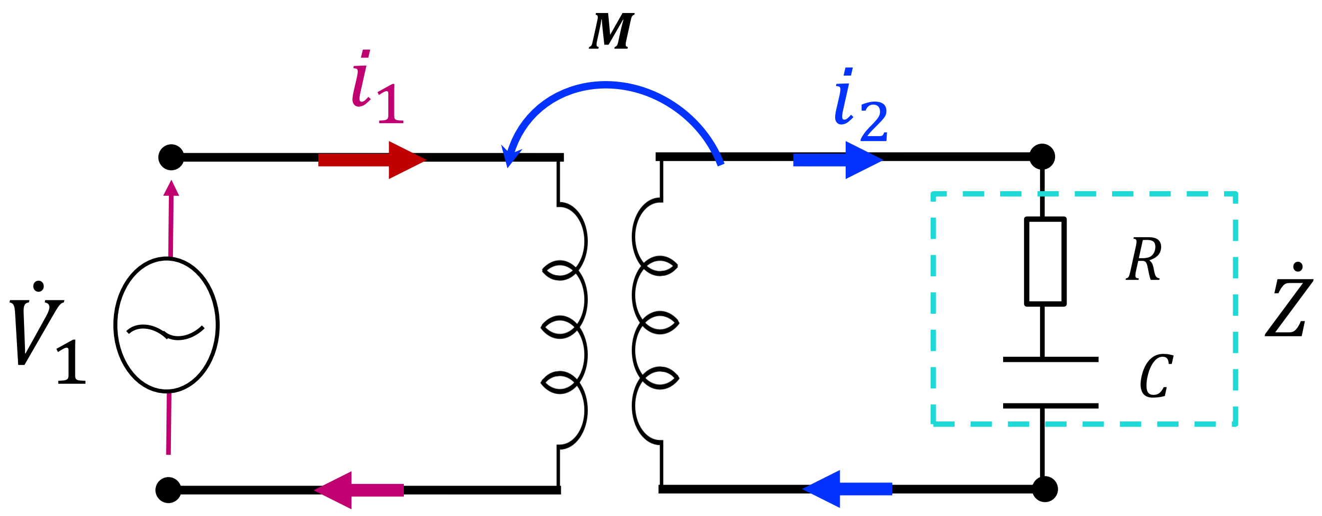
$$\dot{V}_1 = jwL_1 i_1 - jwM \frac{jwM}{jwL_2 + \dot{Z}} i_1$$

$$\dot{V}_1 = \{jwL_1 + \frac{w^2 M^2}{jwL_2 + \dot{Z}}\} i_1$$

$$i_1 = \frac{\dot{V}_1}{jwL_1 + \frac{w^2 M^2}{jwL_2 + \dot{Z}}}$$

下図の電磁誘導結合回路において、L$_{1}$ $=$10[mH], L$_2$=4[mH], M = 2[mH], R =100[Ω], C= 3[μF]、周波数f=500 [Hz]との時、１次回路側（端子a,b）から見たインピーダンス$\dot{Z}_1$の複素数表示と極表示を求めなさい。また、１次回路側に$\dot{V}_1 = 20\angle 0^{o}[V]$ (f=500[Hz])の電圧を加えた時の電流i_1, i_2, 電圧V_2のフェーザ表示を求めなさい。





1次回路

$$\dot{V}_1 = jwL_1 i_1 - jwM_{12}\, i_2$$

2次回路

$$\dot{Z} i_2 = jwM_{21}\, i_1 - jwL_2\, i_2$$

２次回路

$$\dot{Z} i_2 = jwM \ i_1 - jwL_2 \, i_2$$

$$\dot{Z} = R + \frac{1}{jwC}$$

$$i_2 = \frac{jwM}{R + \frac{1}{jwC} + jwL_2} i_1$$

$$i_2 = \frac{jwM}{R + j(wL_2 - \frac{1}{wC})} i_1$$

１次回路

$$\dot{V}_1 = jwL_1 i_1 - jwM i_2$$

$$\dot{V}_1 = jwL_1 i_1 - jwM \boxed{\frac{jwM}{R + j(wL_2 - \frac{1}{wC})} i_1}$$

$$\dot{V}_1 = jwL_1 i_1 + \frac{w^2 M^2}{R + j(wL_2 - \frac{1}{wC})} i_1$$

$$\dot{V}_1 = \left(jwL_1 + \frac{w^2M^2}{R + j(wL_2 - \frac{1}{wC})} \right) i_1$$

$$\dot{Z}_1 = \frac{\dot{V}_1}{i_1} = jwL_1 + \frac{w^2M^2}{R + j(wL_2 - \frac{1}{wC})}$$

これは一次側から見たインピーダンス $\dot{Z}_1$ である。

$$w = 2\pi f = 3140\ (\frac{rad}{s})$$

$L_1 = 1 * 10^{-2}H$　　$M = 2 * 10^{-3}H$

$C = 3 * 10^{-6}F$

$L_2 = 4 * 10^{-3}\ H$　　$R = 100\Omega$

$$\dot{Z}_1 = j * 31.40 + \frac{3140^2 * 4 * 10^{-6}}{100 + j(3.14 * 4 - \frac{1}{3140 * 3 * 10^{-6}})}$$

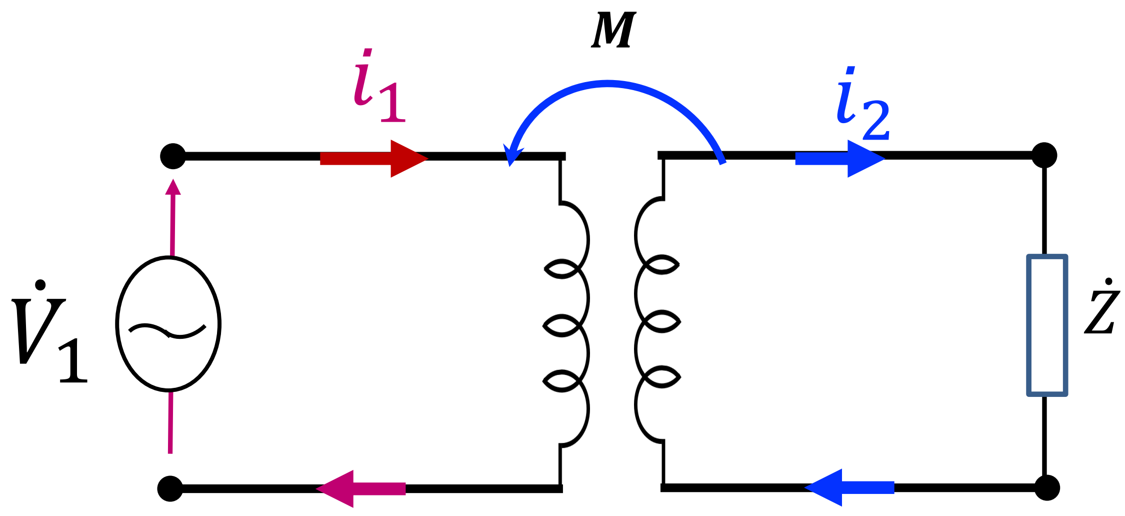
$$\dot{Z}_1 = j * 31.40 + \frac{39.48}{100 - j(93.54)}$$

$$\dot{Z}_1 = j * 31.40 + \frac{39.48 + j3693}{18750} = 0.211 + \mathrm{j}31.7$$

$$\dot{Z}_1 = 0.211 + \mathrm{j}31.7 = \sqrt{0.211^2 + 31.7^2} \angle tan^{-1}(\frac{31.7}{0.211})$$

$$\dot{Z}_1 = 31.7 \angle 89.6^o$$

等価回路概念の導入



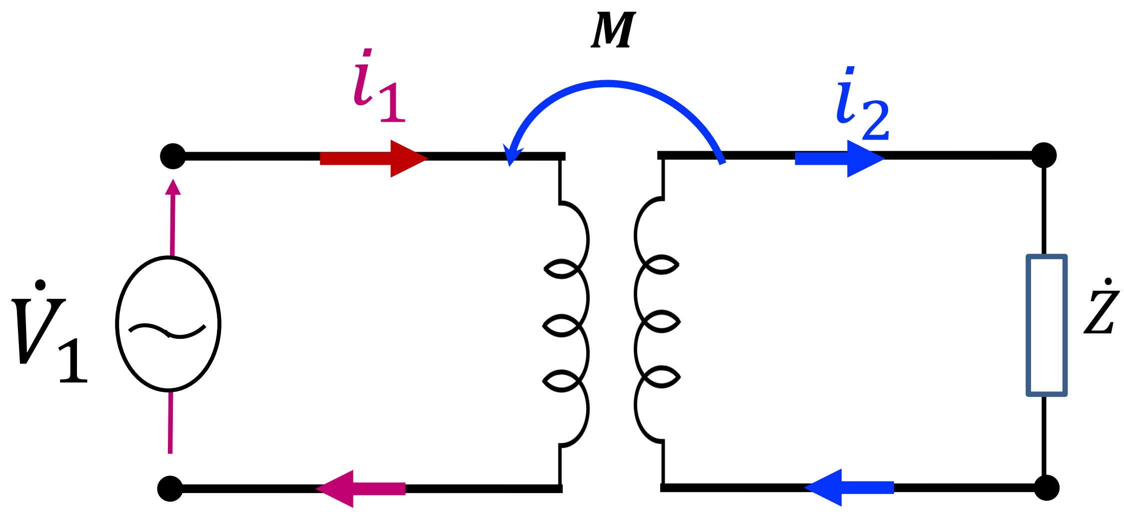
１次回路

$$\dot{V}_1 = jwL_1 i_1 - jw\,M\,i_2$$

２次回路

$$\dot{Z} i_2 = jwM\,i_1 - jwL_2\,i_2$$

等価回路概念の導入



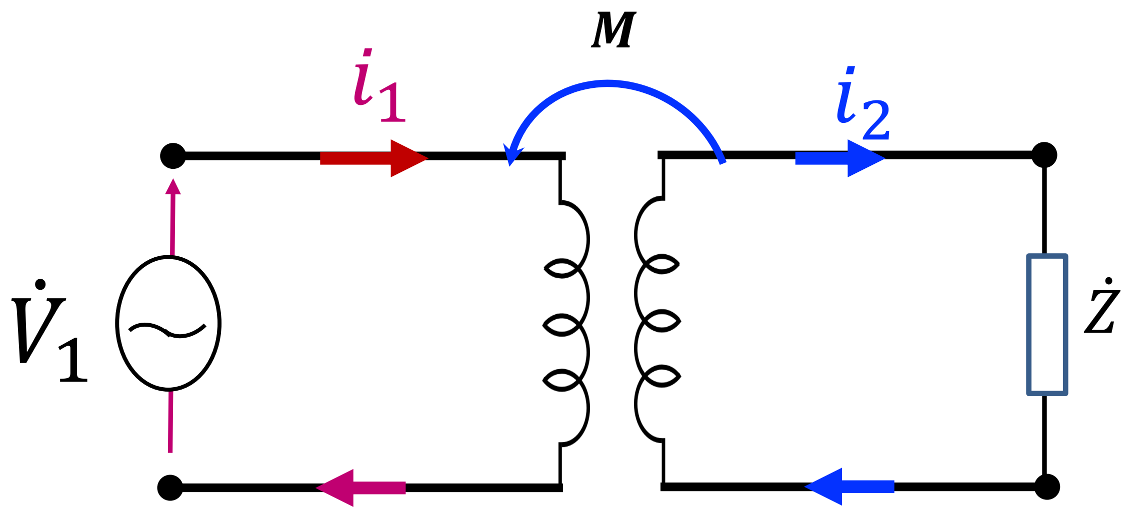
１次回路

$$\dot{V}_1 = \boxed{jwL_1 i_1} - jw\,M\,i_2$$

$$\dot{V}_1 = \boxed{jw(L_1 - M)i_1 + jwMi_1} - jw\,M\,i_2$$

$$\dot{V}_1 = jw(L_1 - M)i_1 + jwM(i_1 - i_2)$$

等価回路概念の導入



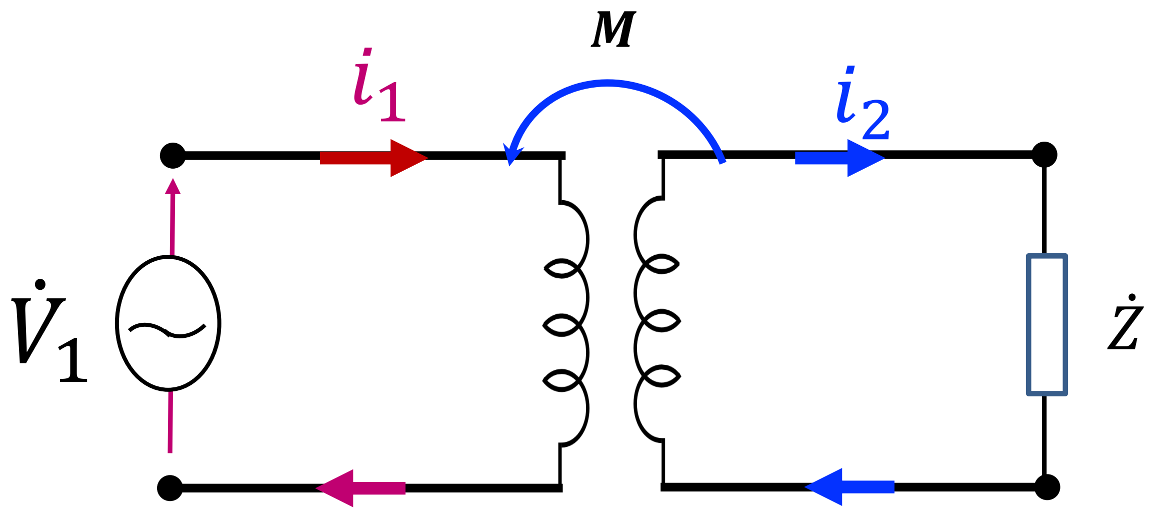
2次回路

$$\dot{Z} i_2 = jwM\, i_1 - \boxed{jwL_2\, i_2}$$

$$\dot{Z} i_2 = jwM\, i_1 - \boxed{\mathrm{jwM}\, i_2 - jw(L_2 - M)\, i_2}$$

$$\dot{Z} i_2 = jwM\,(i_1 - i_2) - jw(L_2 - M)\, i_2$$

等価回路概念の導入



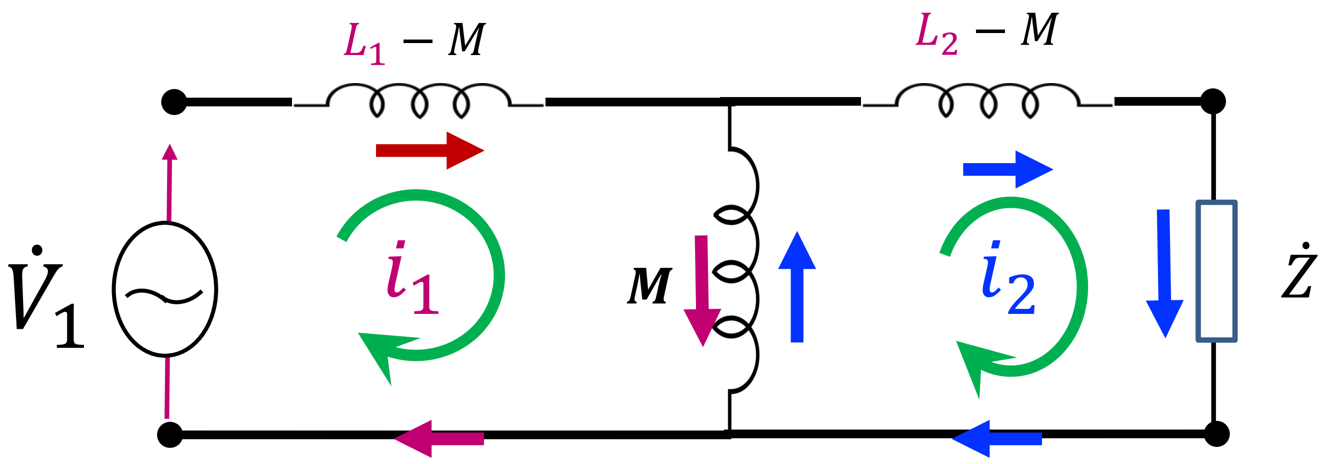
１次回路

$$\dot{V}_1 = jw(L_1 - M)i_1 + jwM(i_1 - i_2)$$

２次回路

$$\dot{Z} i_2 = jwM\,(i_1 - i_2) - jw(L_2 - M)\,i_2$$

等価回路概念の導入



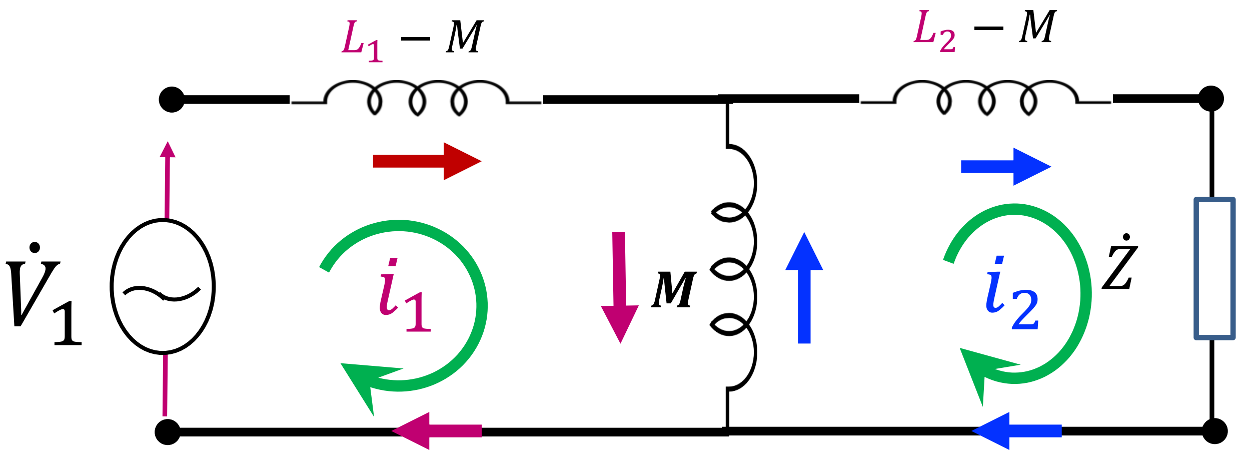
１次回路

$$\dot{V}_1 = jw(L_1 - M)i_1 + jwM(i_1 - i_2)$$

２次回路

$$\dot{Z}i_2 = jwM\,(i_1 - i_2) - jw(L_2 - M)\,i_2$$

等価回路概念の導入



1次回路

$$\dot{V}_1 = jw(L_1 - M)i_1 + jwM(i_1 - i_2)$$

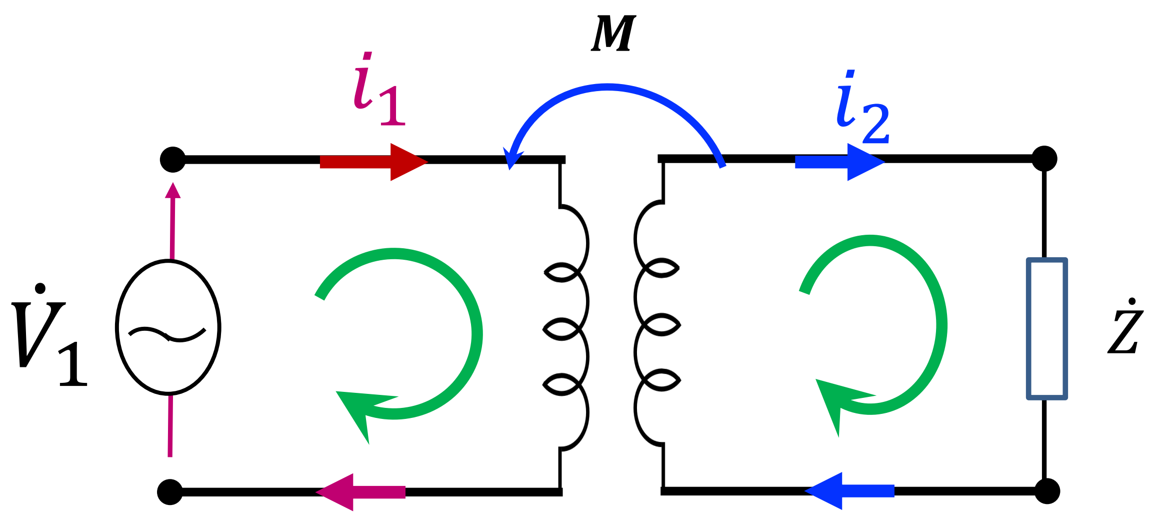
$$-\dot{V}_1 + jw(L_1 - M)i_1 + jwM(i_1 - i_2) = 0$$

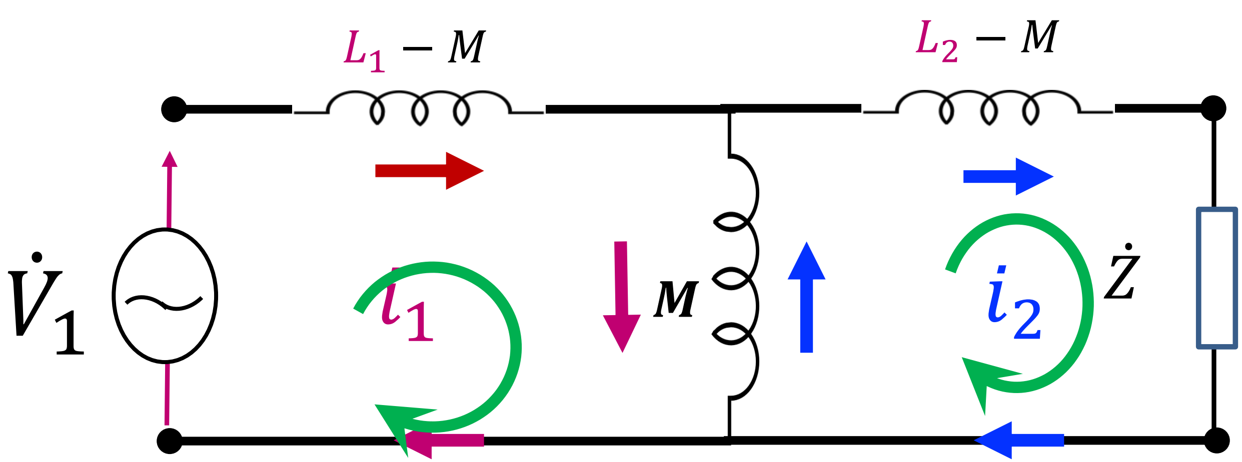
2次回路

$$-\dot{Z}i_2 = jwM\,(i_2 - i_1) + jw(L_2 - M)\,i_2$$

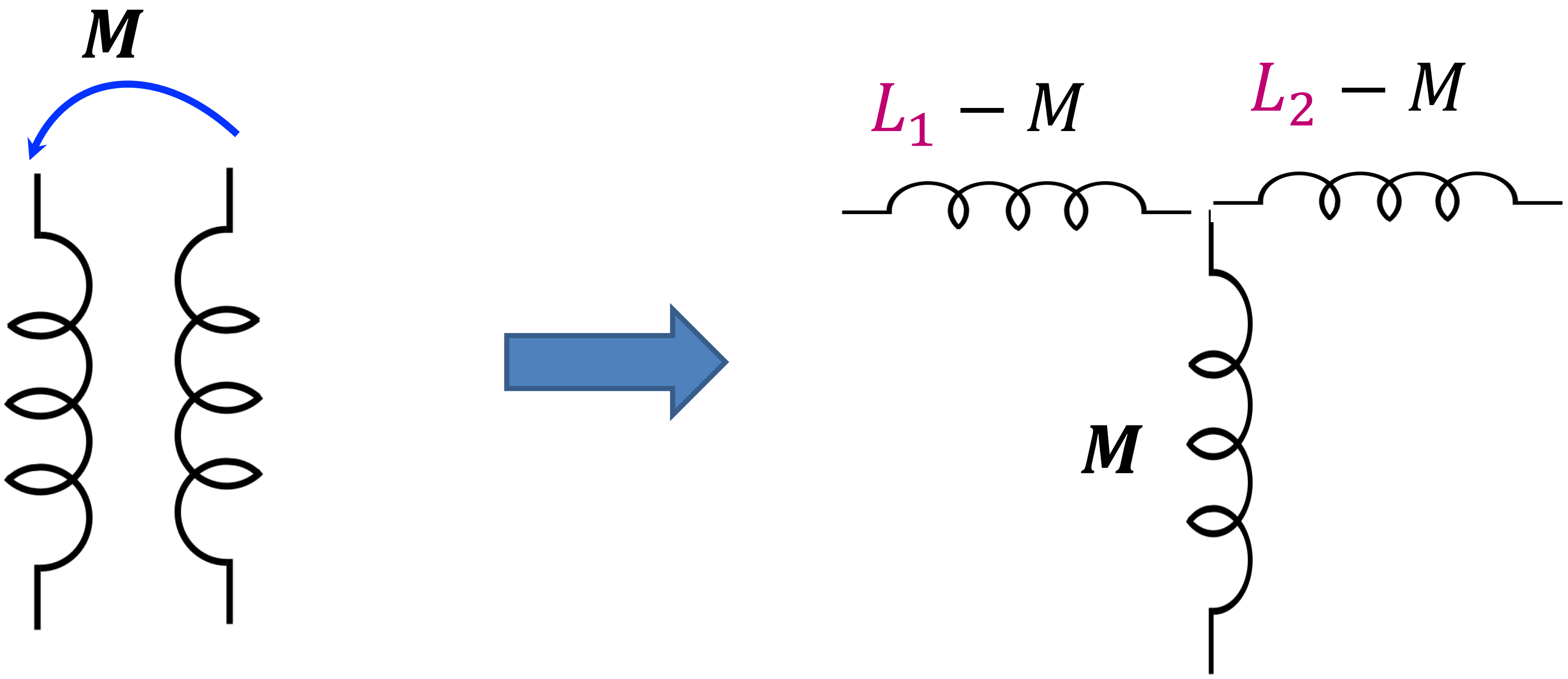
$$\dot{Z}i_2 + jwM\,(i_2 - i_1) + jw(L_2 - M)\,i_2 = 0$$

等価回路概念の導入

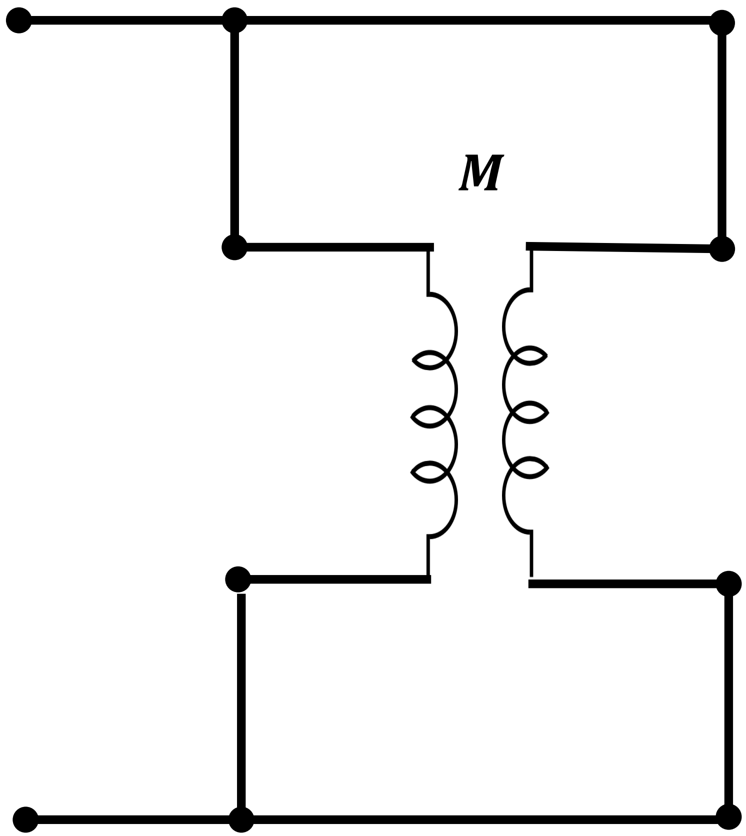


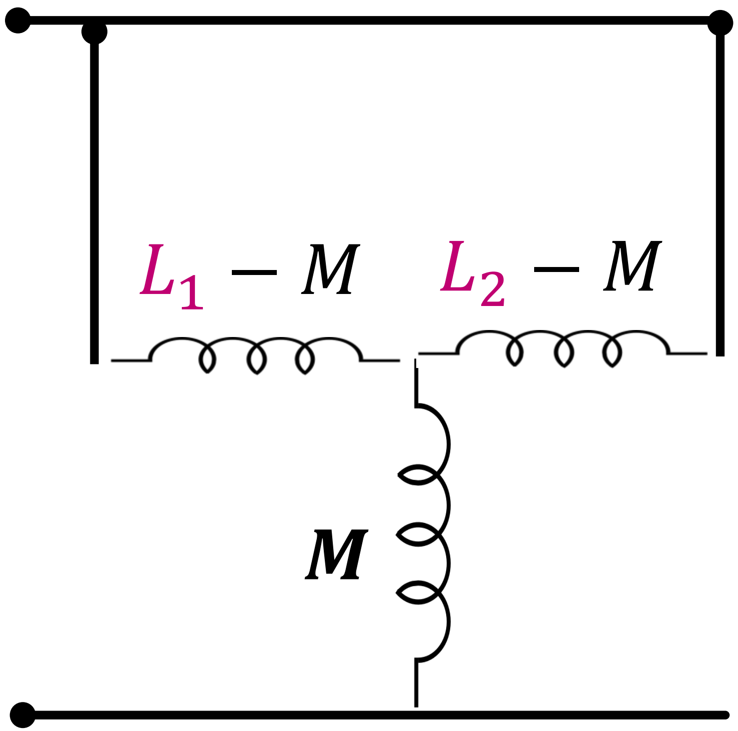


等価回路概念の導入

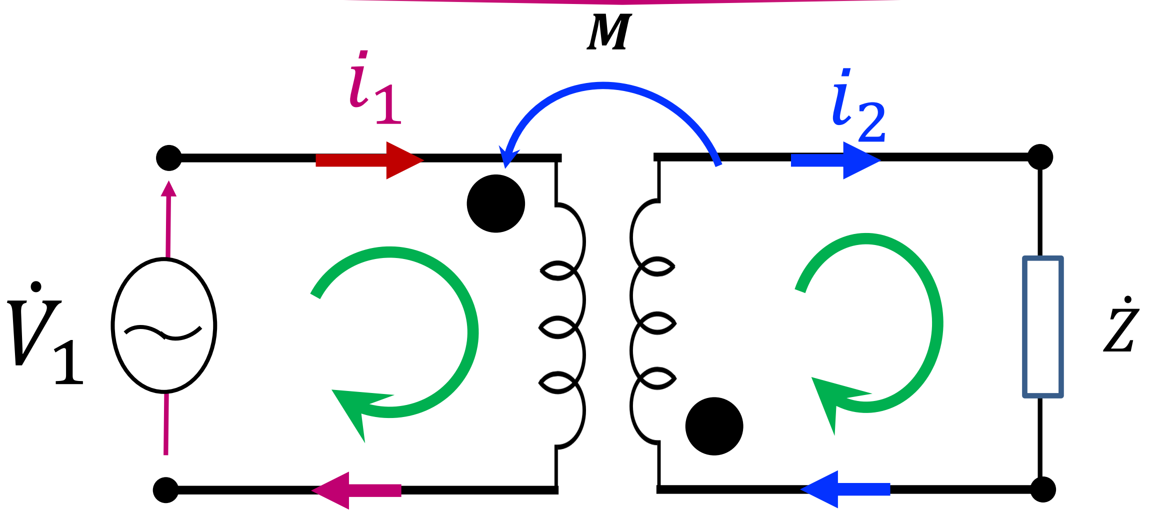


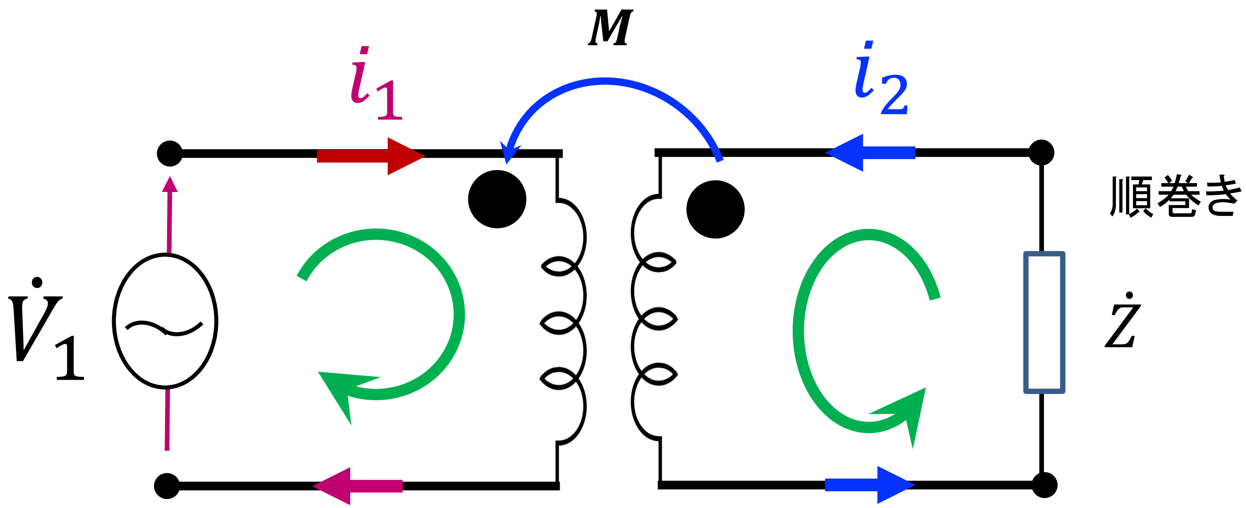
等価回路の例:





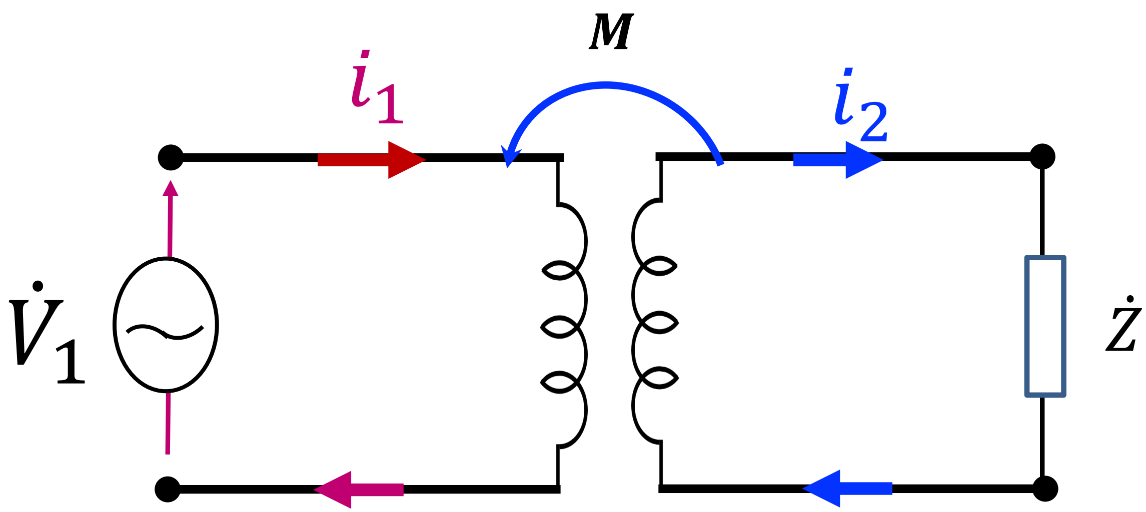
電流の方向・順巻き・逆巻き：ドット表記





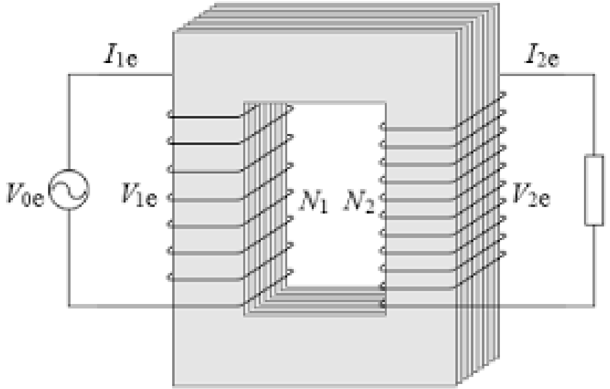
変圧器と理想変圧器

相互インダクタンスM:結合係数



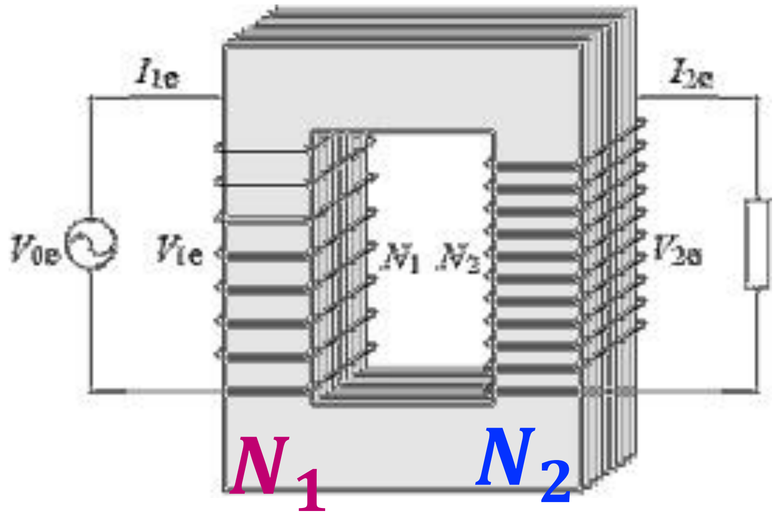
$$\boldsymbol{M = k\sqrt{L_1 L_2}}$$

$\boldsymbol{0 < k \leq 1}$　結合係数



$\boldsymbol{k = 1}$：変圧器結合

変圧器結合



$$\frac{N_1}{N_2} = n$$

$$M = \sqrt{L_1 L_2}$$

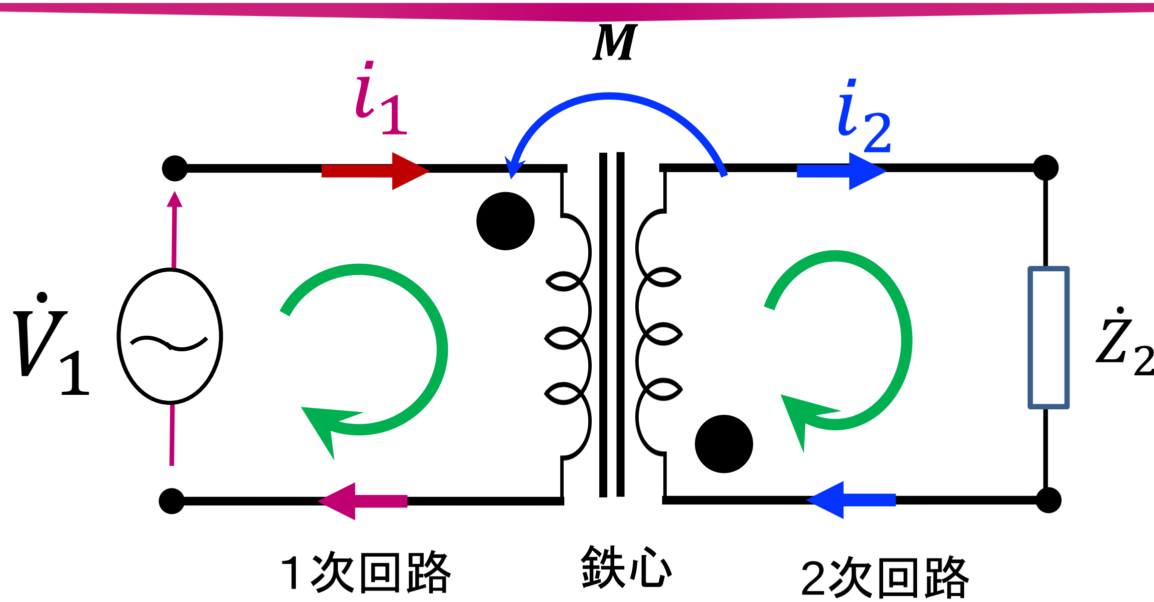
$$L_1 = A * {N_1}^2$$

$$L_2 = A * {N_2}^2$$

$$M = A N_1 N_2$$

$$\frac{L_1}{L_2} = \frac{{N_1}^2}{{N_2}^2} = n^2$$

変圧器結合回路:インピーダンス



$$\dot{Z}_1 = jwL_1 + \frac{w^2M^2}{jwL_2 + \dot{Z}_2}$$

$$\dot{Z}_1 = \frac{-w^2\boldsymbol{L_1L_2} + \boldsymbol{jwL_1}\dot{Z}_2 + w^2M^2}{jwL_2 + \dot{Z}_2}$$

変圧器結合回路:インピーダンス

$$\boldsymbol{M} = \sqrt{\boldsymbol{L_1 L_2}}\text{を代入する}$$

$$\dot{Z}_1 = \frac{-w^2 \boldsymbol{L_1 L_2} + \boldsymbol{jwL_1}\dot{Z}_2 + w^2 \boldsymbol{L_1 L_2}}{jwL_2 + \dot{Z}_2}$$

$$\dot{Z}_1 = \frac{\boldsymbol{jwL_1}\dot{Z}_2}{jwL_2 + \dot{Z}_2}$$

$\boldsymbol{jwL_1}\dot{Z}_2$

$$\dot{Y}_1 = \frac{1}{\dot{Z}_1} = \frac{jwL_2 + \dot{Z}_2}{\boldsymbol{jwL_1}\dot{Z}_2}$$

変圧器結合回路:インピーダンスとアドミタンス

$$\dot{Y}_1 = \frac{1}{(\frac{L_1}{L_2})\dot{Z}_2} + \frac{1}{jwL_1}$$

$$\dot{Y}_1 = \frac{1}{n^2\dot{Z}_2} + \frac{1}{jwL_1}$$

$$\dot{Z}_1 = (\frac{1}{n^2\dot{Z}_2} + \frac{1}{jwL_1})^{-1}$$

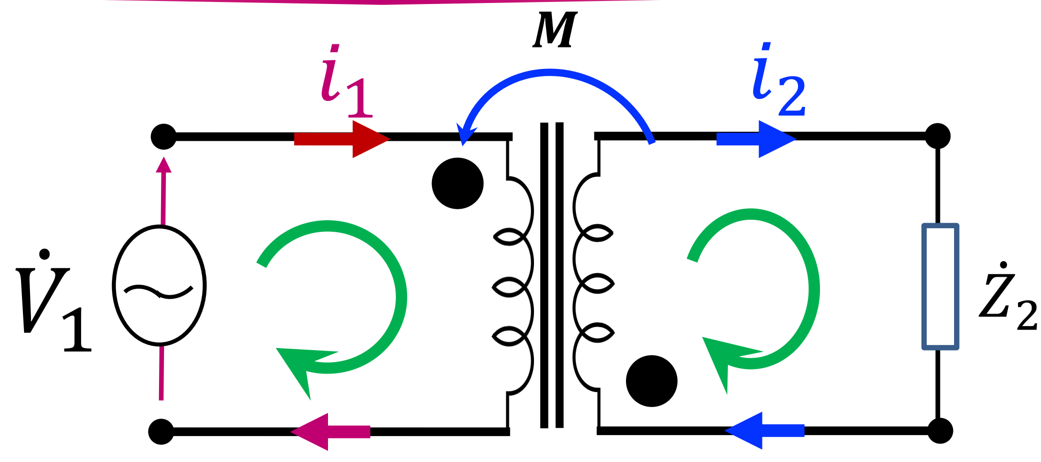
$$\dot{Z}_1 = jwL_1 + \frac{w^2 M^2}{jwL_2 + \dot{Z}}$$

$$M = \sqrt{\boldsymbol{L_1 L_2}}$$

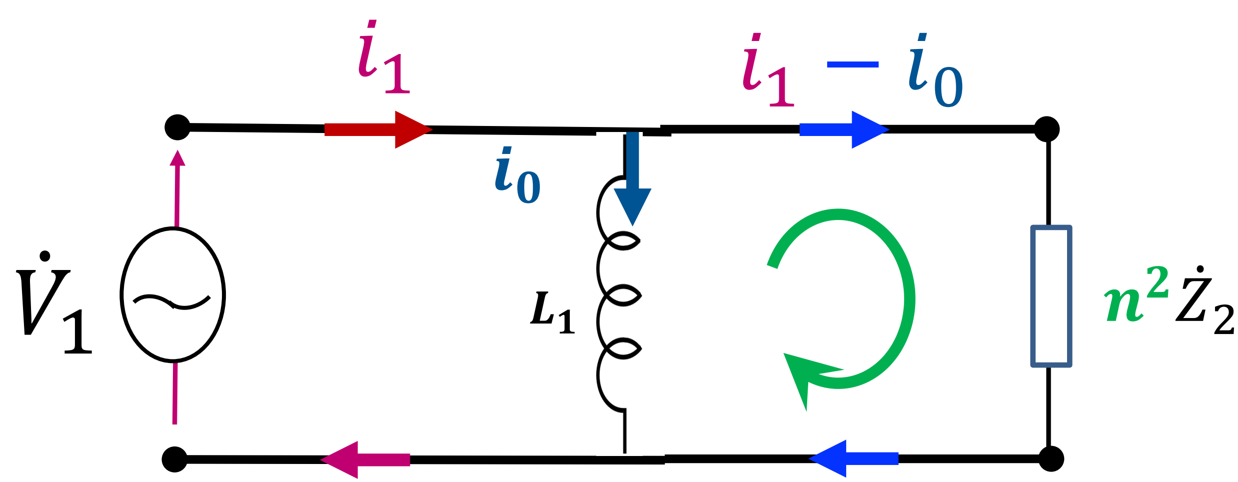
$$\frac{\boldsymbol{L_1}}{\boldsymbol{L_2}} = \frac{\boldsymbol{N_1}^2}{\boldsymbol{N_2}^2} = \boldsymbol{n}^2$$

$$\dot{Z}_1 = \left(\frac{1}{\boldsymbol{n}^2 \dot{Z}_2} + \frac{1}{\boldsymbol{jwL_1}}\right)^{-1}$$

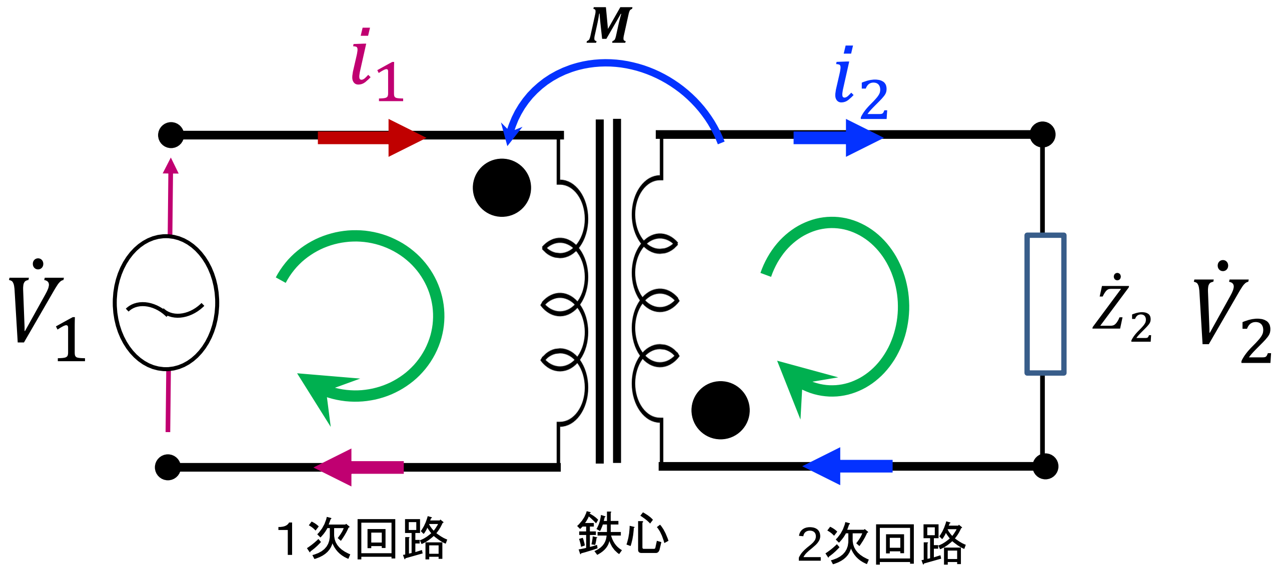
変圧器結合回路：等価回路



等価回路



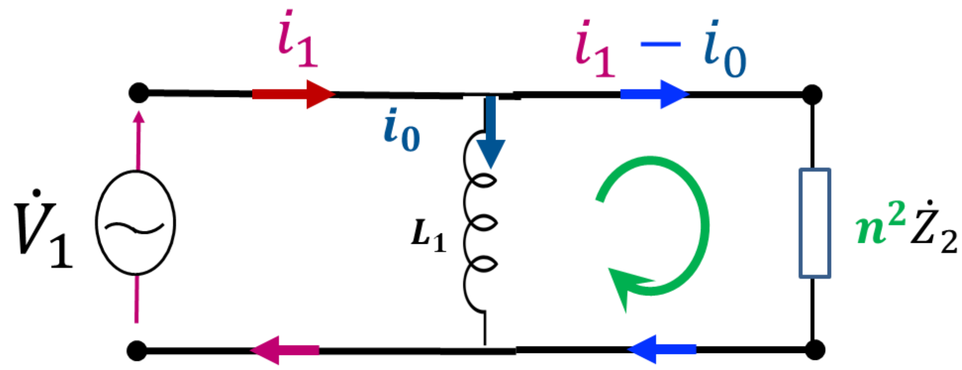
変圧器結合回路：電圧関係



$$i_2 = \frac{jwM}{jwL_2 + \dot{Z}_2} i_1$$

$$\dot{V}_2 = \dot{Z}_2 \, i_2$$

$$i_2 = \frac{jwM}{jwL_2 + \dot{Z}_2} i_1$$



$$\dot{Y}_1 = \frac{1}{n^2\dot{Z}_2} + \frac{1}{jwL_1}$$

$$i_1 = \frac{jwL_2 + \dot{Z}_2}{jwL_1\dot{Z}_2}\dot{V}_1$$

$$i_2 = \frac{jwM}{jwL_2 + \dot{Z}_2} * \frac{jwL_2 + \dot{Z}_2}{jwL_1\dot{Z}_2}\dot{V}_1$$

$$i_2 = \frac{jwM}{\boldsymbol{jwL_1}\dot{Z}_2}\dot{V}_1$$

$$\dot{V}_2 = \dot{Z}_2\ i_2$$

$$\dot{V}_2 = \dot{Z}_2 \frac{jwM\ \dot{V}_1}{wL_1\dot{Z}_2}$$

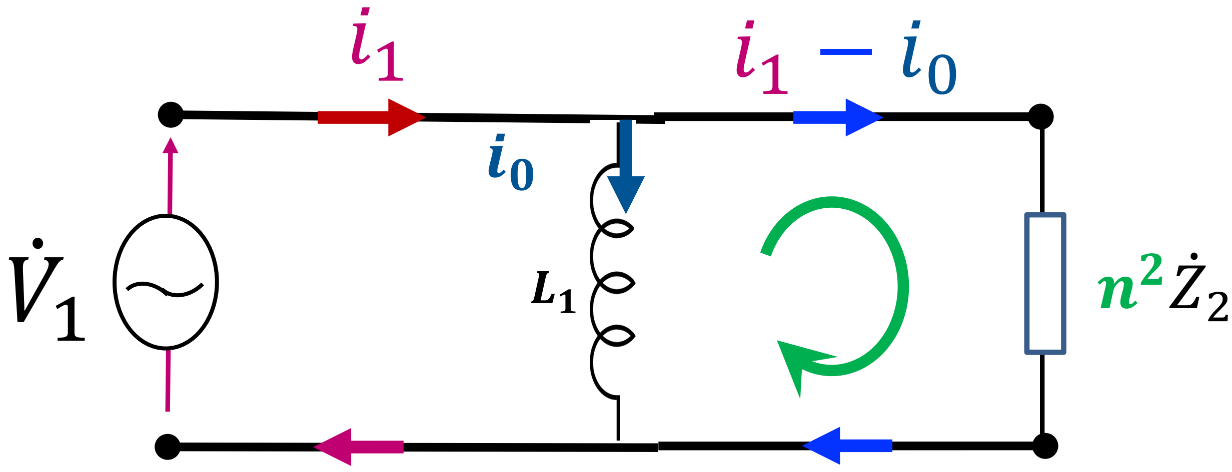
変圧器結合回路:電圧関係

$$\dot{V}_2 = \cancel{\dot{Z}_2}\frac{\cancel{jw}\sqrt{\boldsymbol{L_1 L_2}}\,\dot{V}_1}{\cancel{jw}L_1\cancel{\dot{Z}_2}}$$

$$\dot{V}_2 = \sqrt{\frac{\boldsymbol{L_2}}{L_1}}\,\dot{V}_1 = \frac{\boldsymbol{N_2}}{N_1}\,\dot{V}_1 = \frac{\boldsymbol{1}}{n}\dot{V}_1$$

暗記: $\dot{V}_1 = n\ \dot{V}_2$

変圧器結合回路:等価回路



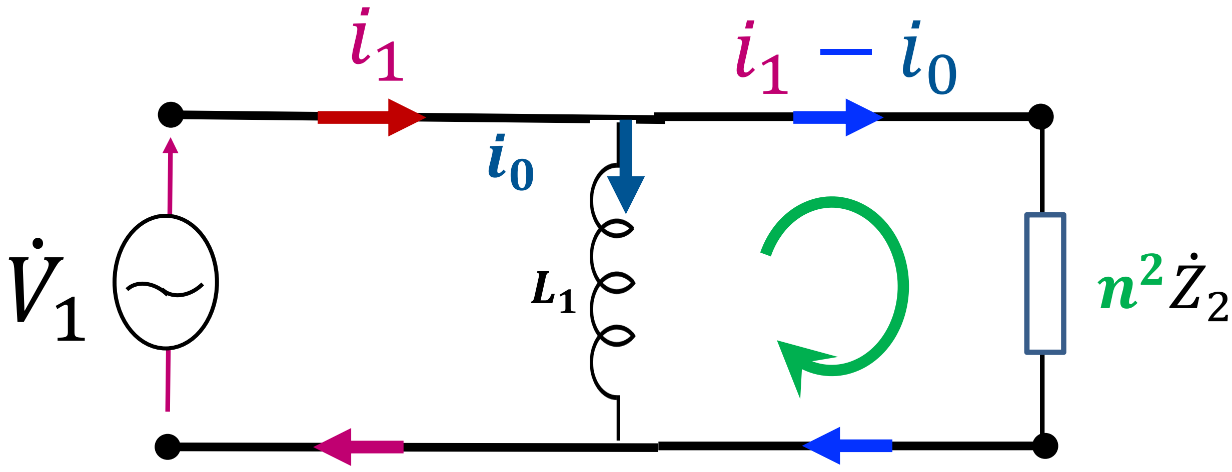
$$\frac{1}{\dot{Z}_1} = \frac{1}{n^2\dot{Z}_2} + \frac{1}{jwL_1}$$

通常、変圧器は自己インダクタンスL_1を十分大きくして、$wL_1 \gg n^2\dot{Z}_2$

$$\dot{Z}_1 = n^2\dot{Z}_2$$

理想変圧器

変圧器結合回路:等価回路



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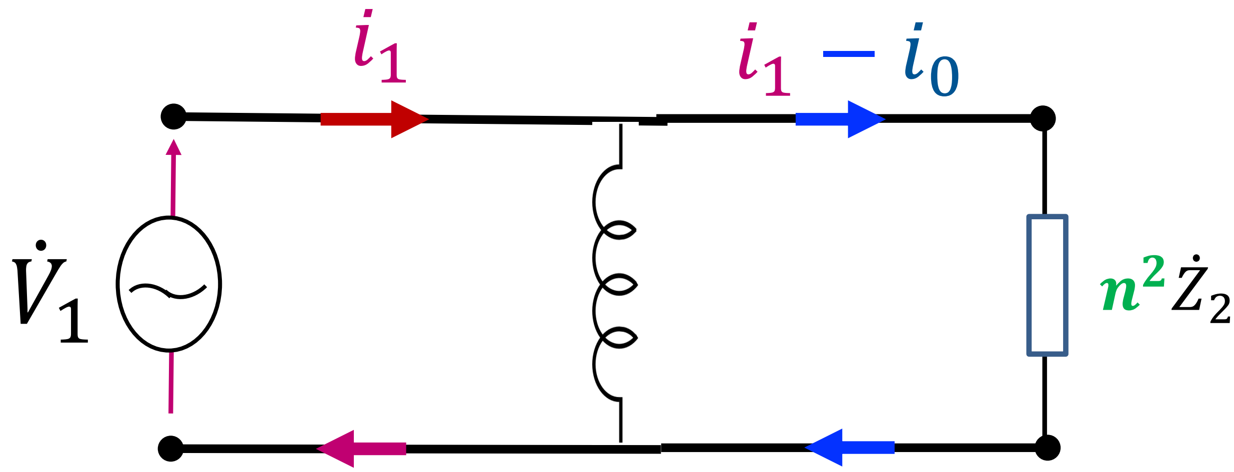
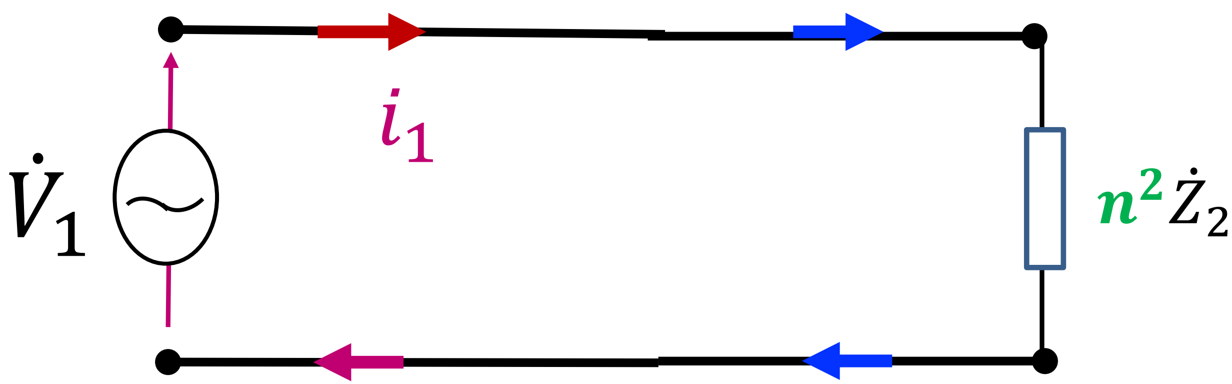
理想変圧器

理想变压器:

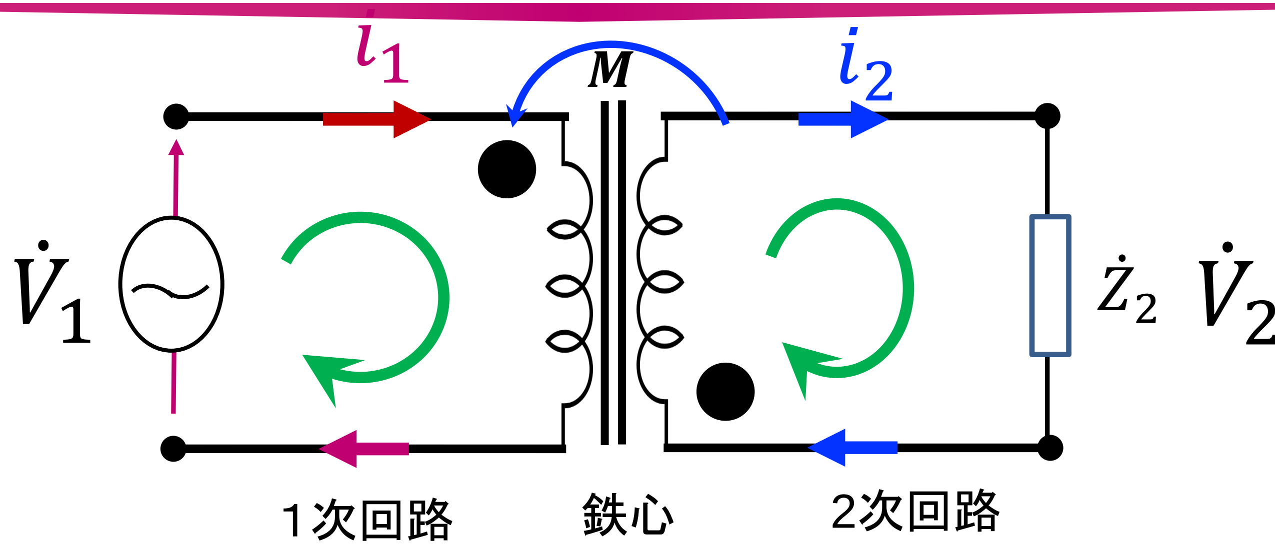


理想回路

$$M = \sqrt{L_1 L_2} \qquad \dot{Z}_1 = n^2 \dot{Z}_2$$



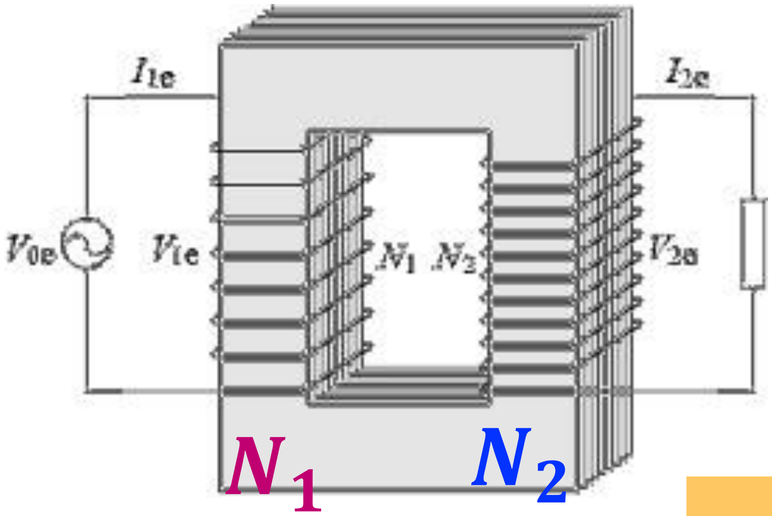
変圧器結合回路：電流関係　理想変圧器



$$\dot{Z}_1 = n^2 \dot{Z}_2$$

$$i_2 = \frac{\dot{V}_2}{\dot{Z}_2} = \frac{\frac{1}{n}\dot{V}_1}{\dot{Z}_2} = \frac{\frac{1}{n}\dot{V}_1}{\frac{1}{n^2}\dot{Z}_1} = n\, i_1$$

理想变压器



$$\frac{N_1}{N_2} = n$$

暗記:

$$i_2 = n\, i_1$$

$$\dot{V}_1 = n\, \dot{V}_2$$