

# 基礎電氣回路CH-7

# キルヒホッフ電流・電圧法則：回路解析

## 直流回路

$$V = IR$$



時間応答・過渡現象

古典法  
ラプラス法

## 交流回路

$$v(t) = \sqrt{2}V_e \sin(\omega t - \theta)$$

複素数とフェーザ表示



周波数応答

周波数一定

直列共振回路  
並列共振回路  
相互誘導回路

交流のオーム法則  
交流電力  
交流回路網

# 交流回路の周波数特性

$$v(t) = \sqrt{2}V_e \sin(\omega t - \theta)$$

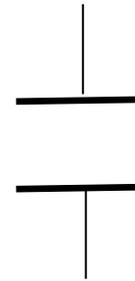
$R \angle 0$



$j\omega L = \omega L \angle 90^\circ$



$\frac{1}{j\omega C} = \frac{1}{\omega C} \angle -90^\circ$



周波数特性:  $f$

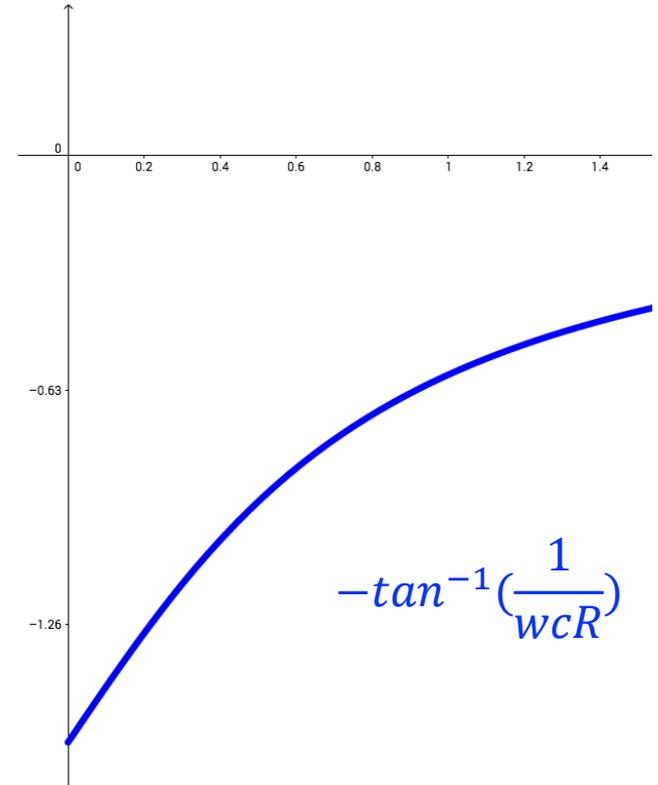
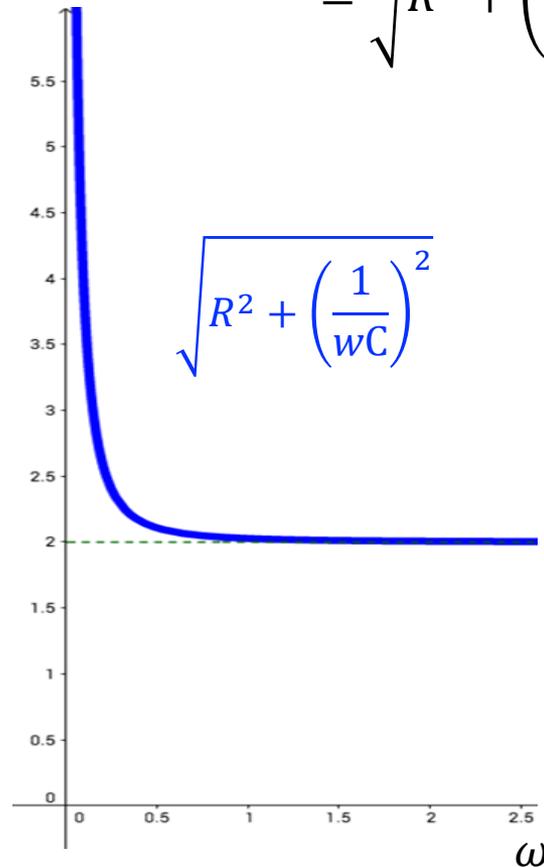
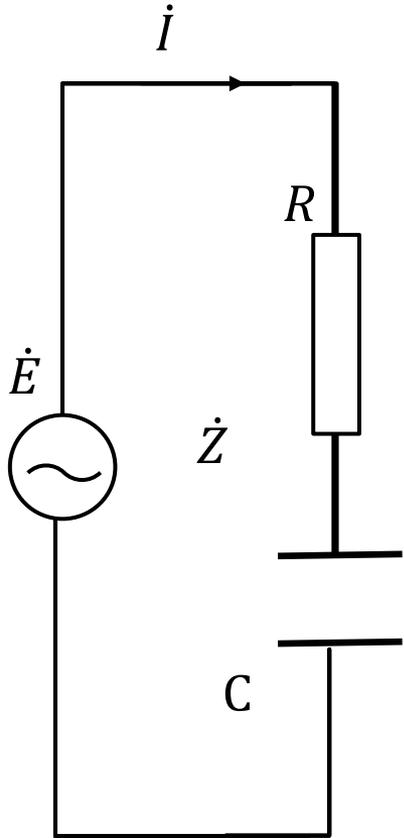
角周波数特性:  $\omega = 2\pi f$

# 抵抗RとキャパシタンスCの直列回路の周波数特性

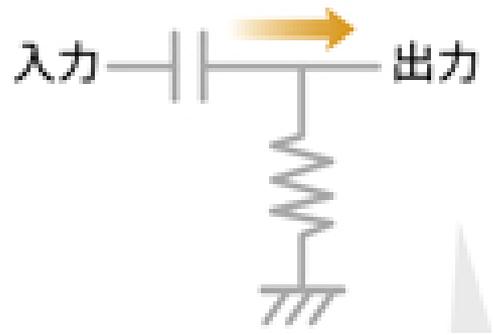
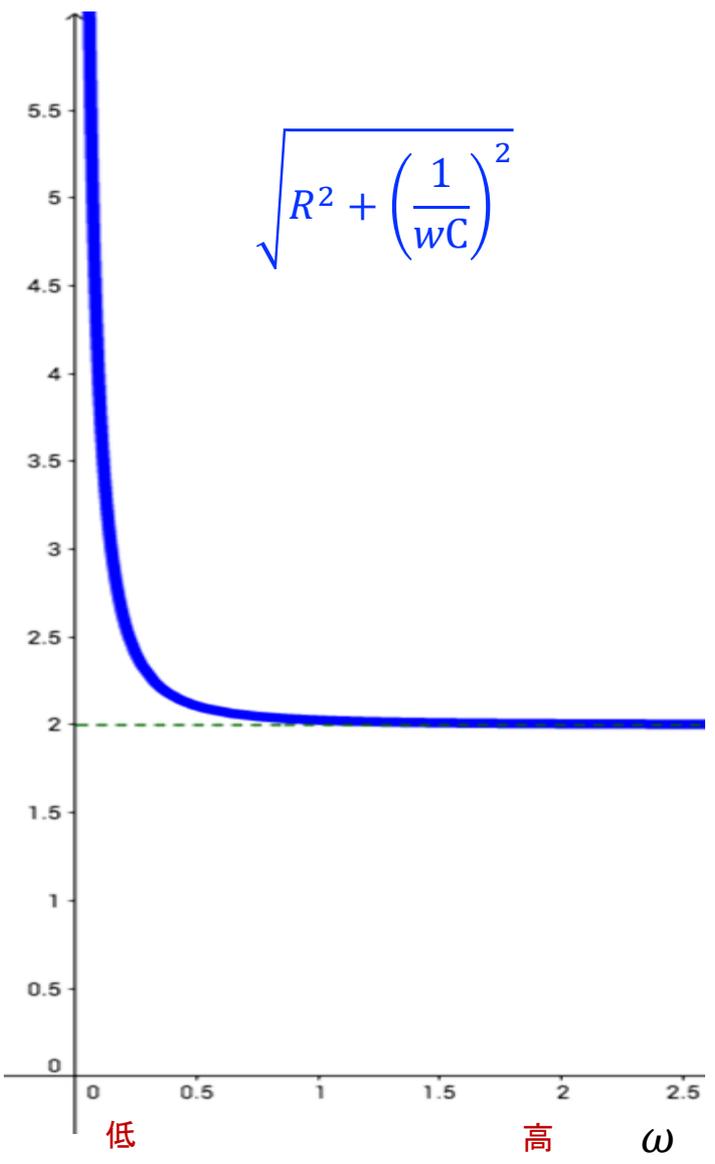
$$\text{インピーダンス: } \dot{Z} = R + \frac{1}{j\omega C}$$

$$\dot{Z} = R - j\frac{1}{\omega C}$$

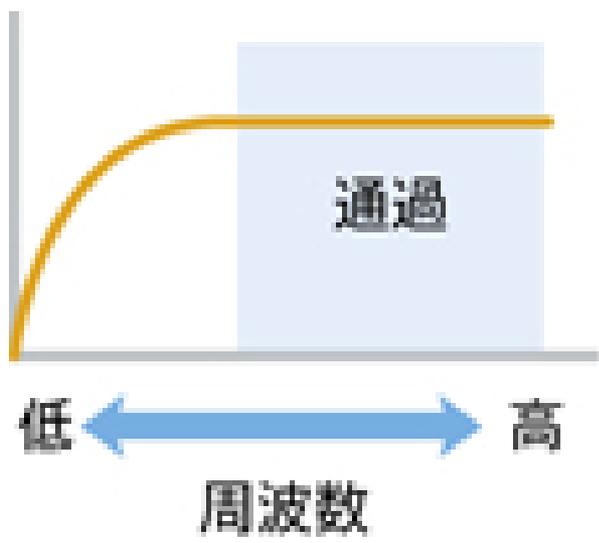
$$= \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2} \angle -\tan^{-1}\left(\frac{1}{\omega CR}\right)$$



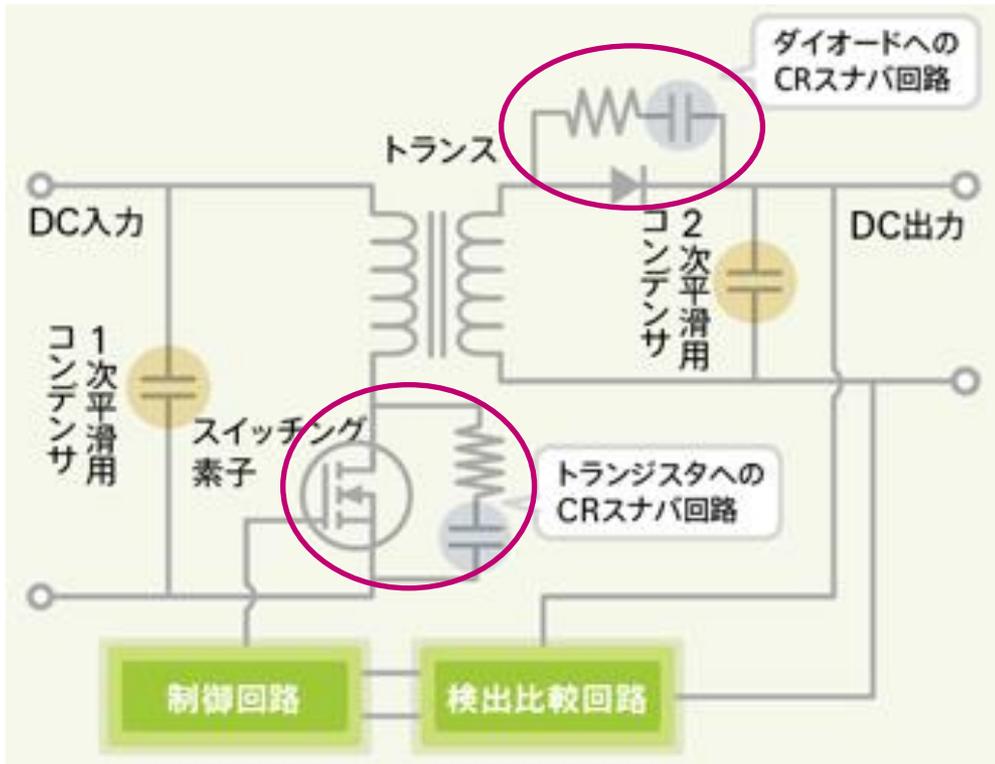
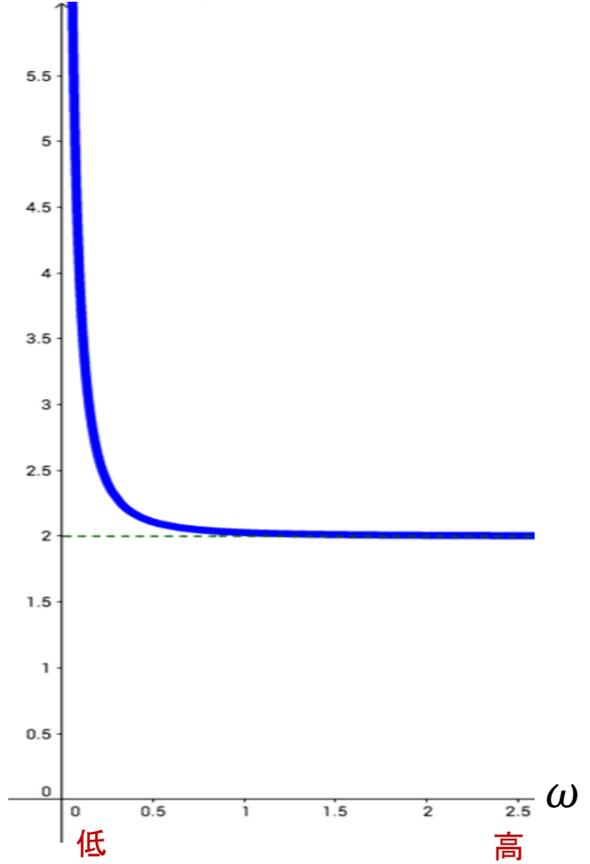
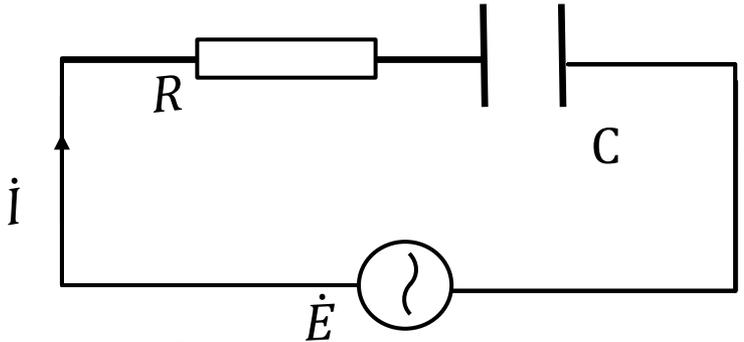
# 直列R-C回路の応用:ハイパスフィルタ



低い周波数成分を遮断し、高い周波数成分だけを通過させて出力する

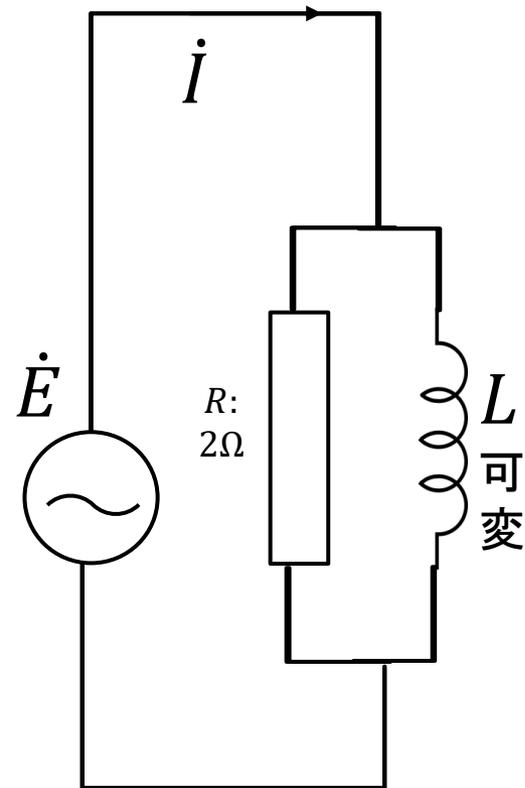


# 直列R-C回路の応用:ハイパスフィルタ



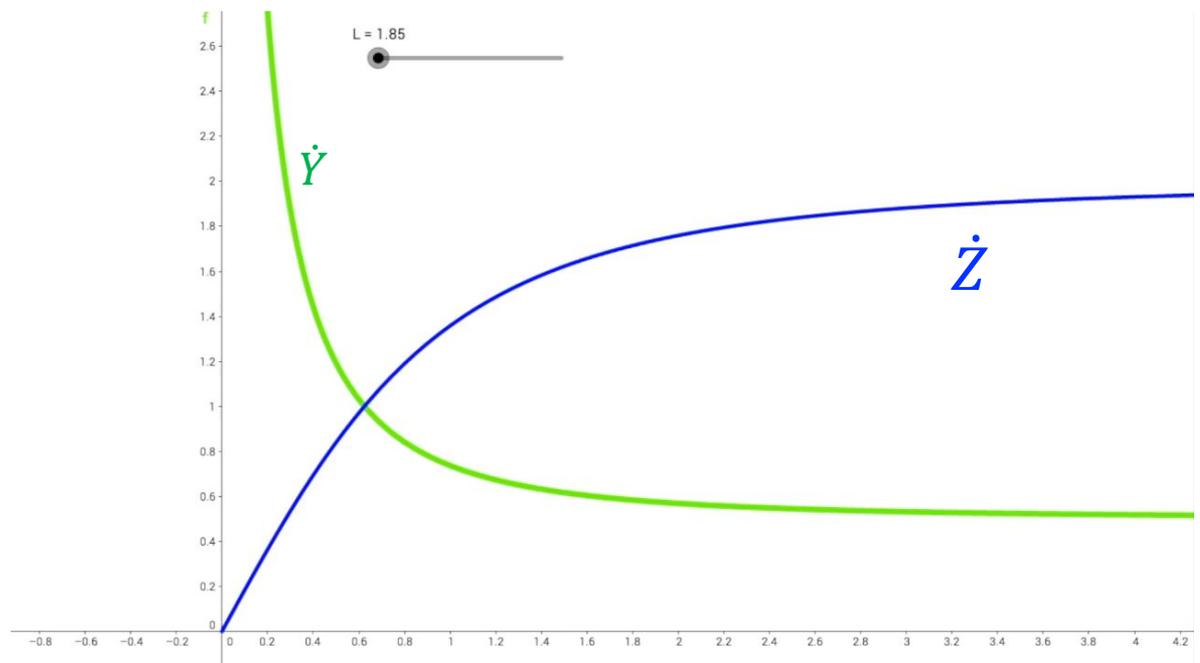
ノイズ対策にも不可欠な部品  
DC-DCコンバータ

# 抵抗RとコイルLの並列回路の周波数特性



$$\dot{Y} = \frac{1}{\dot{Z}} = \frac{1}{R} + \frac{1}{j\omega L} \quad Y = \frac{1}{R} - j \frac{1}{\omega L}$$

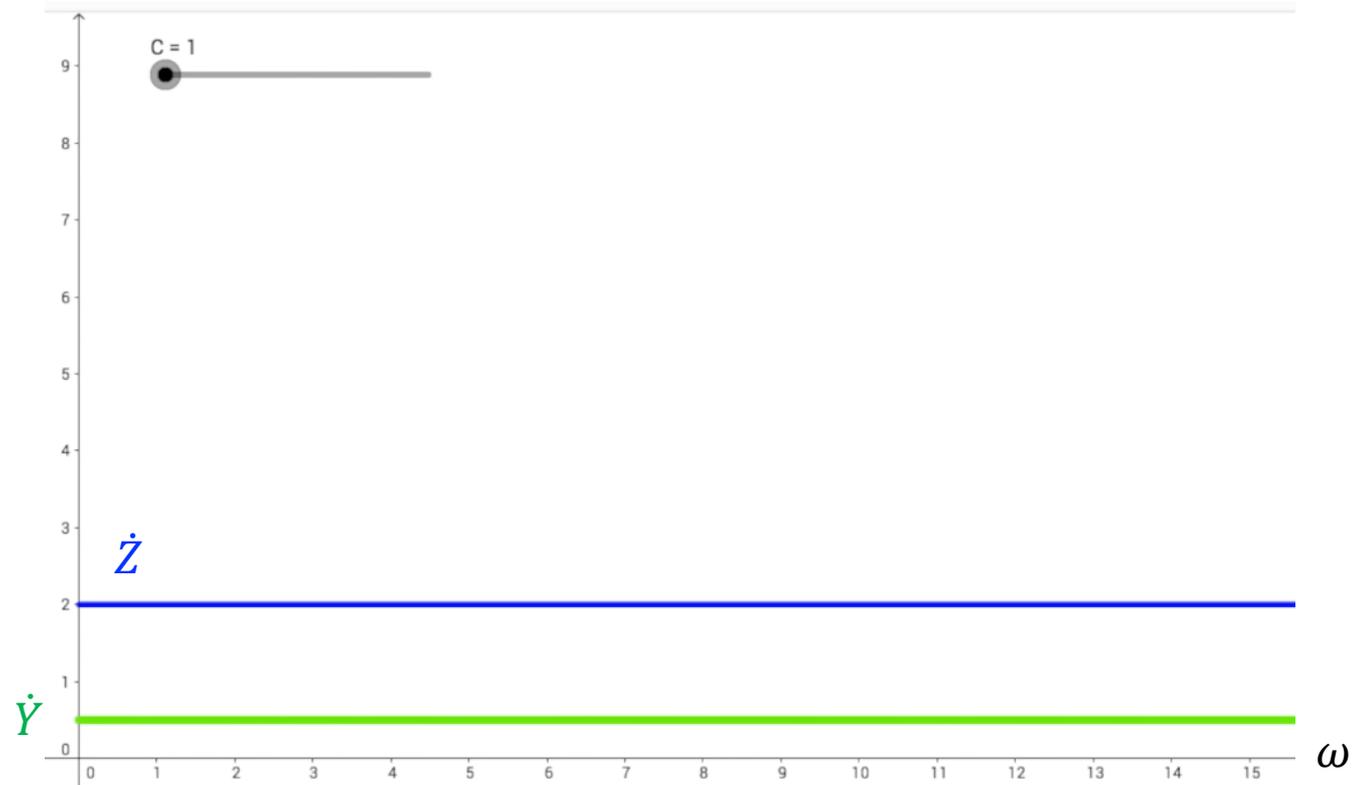
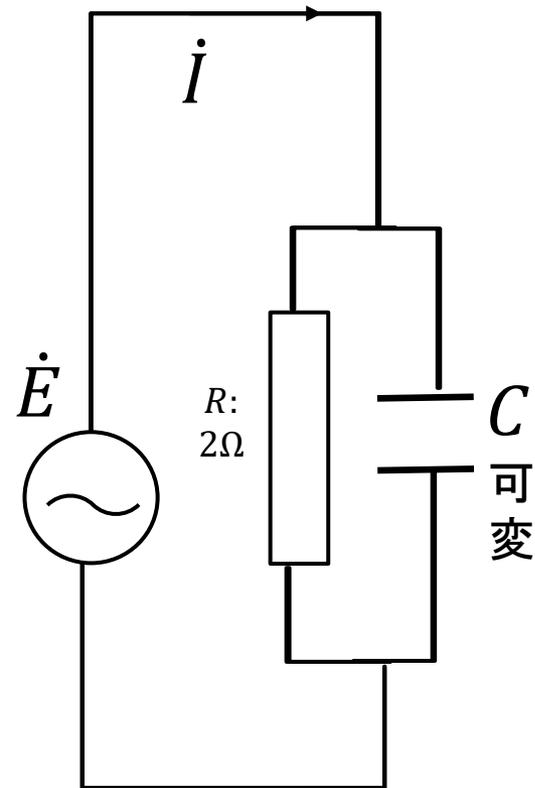
$$Y = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{\omega L}\right)^2} \quad \angle -\tan^{-1}\left(\frac{R}{\omega L}\right)$$



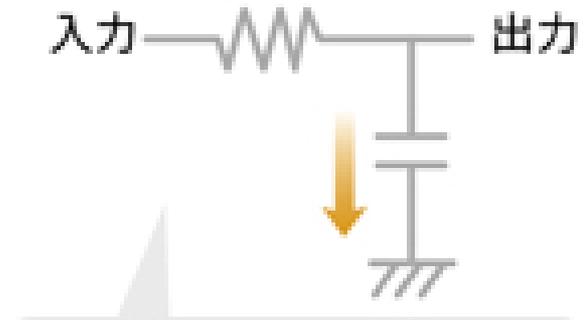
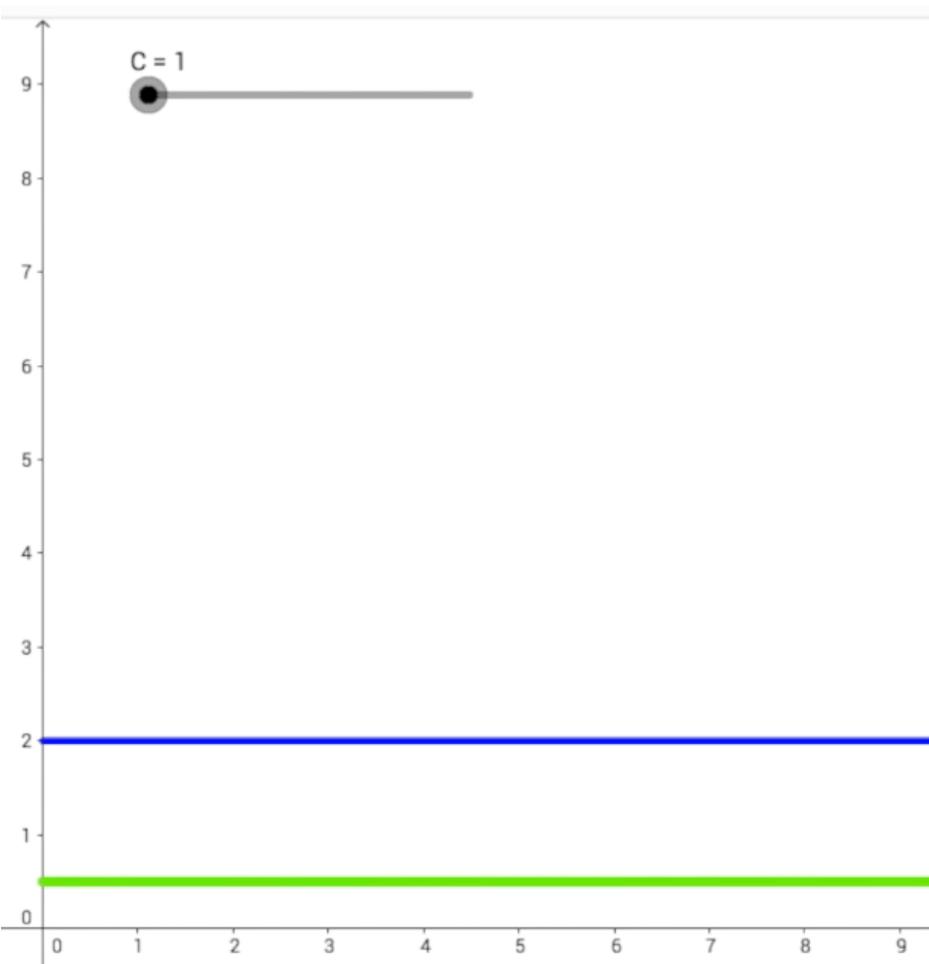
# 抵抗RとコンデンサCの並列回路の周波数特性

$$\dot{Y} = \frac{1}{\dot{Z}} = \frac{1}{R} + j\omega C$$

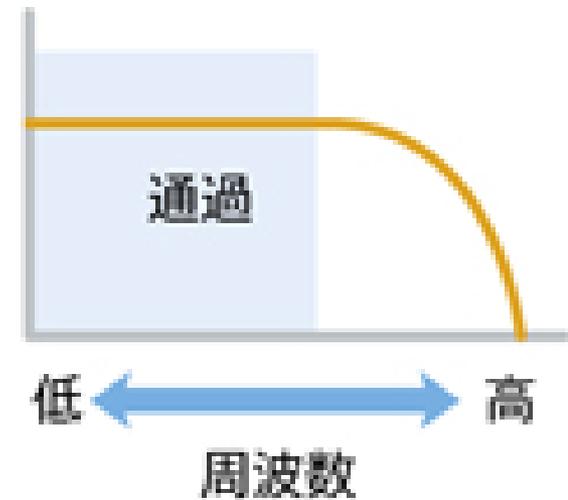
$$\dot{Y} = \sqrt{\left(\frac{1}{R}\right)^2 + (\omega C)^2} \quad \angle \tan^{-1}(\omega CR)$$



# 並列R-C回路の応用: ローパスフィルタ



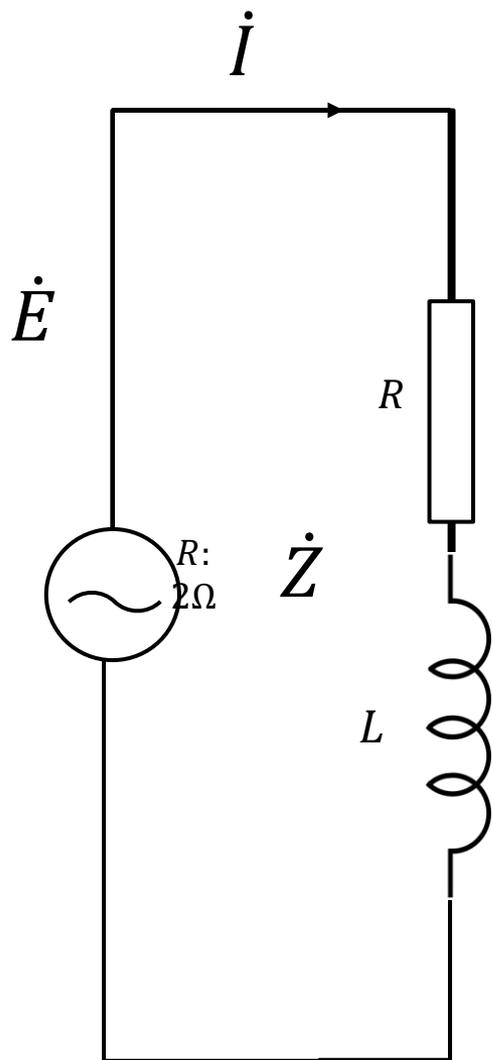
高い周波数成分をグランドに逃し、  
低い周波数成分だけを通過させ  
て出力する





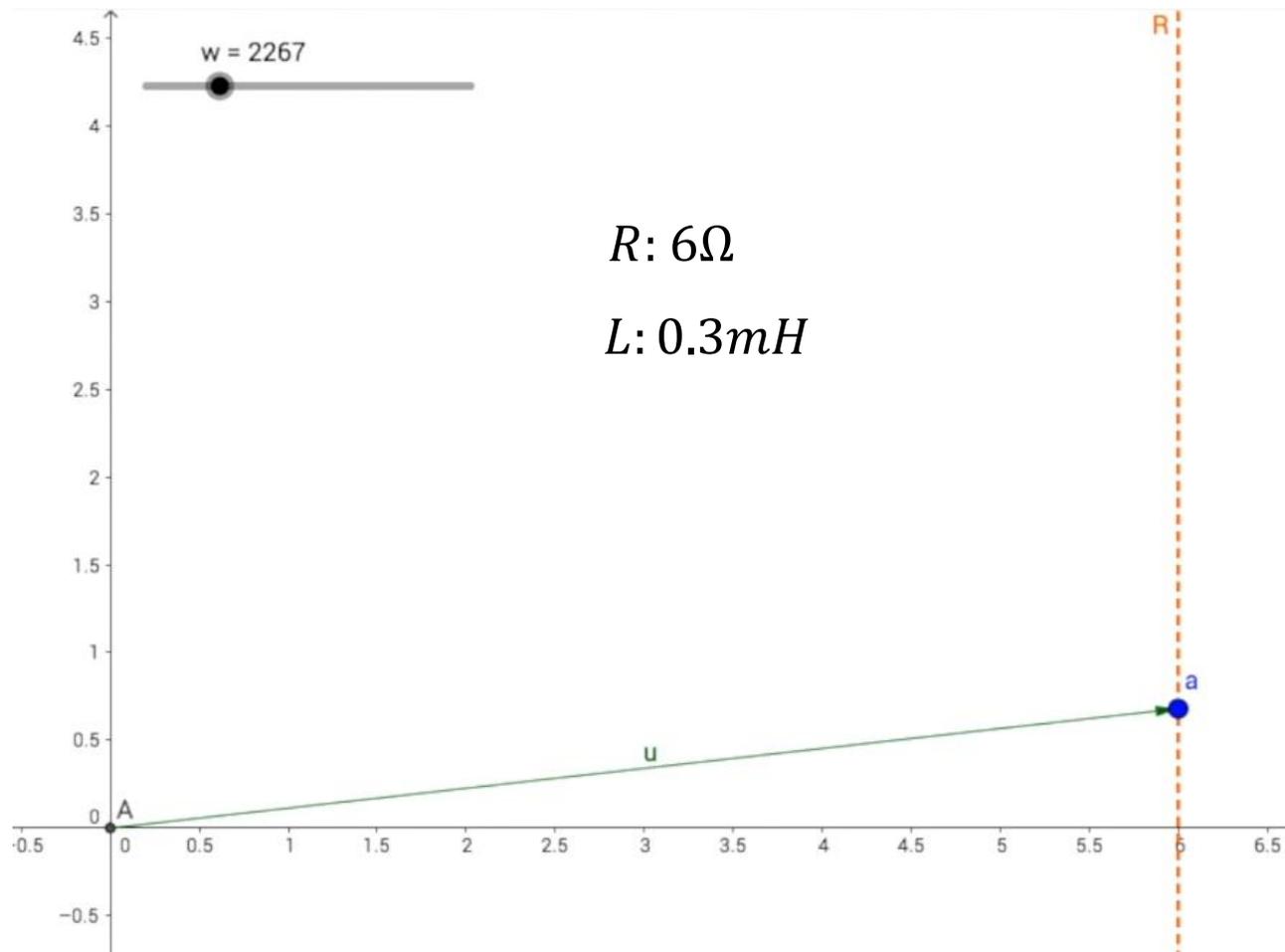


# R-L直列回路のインピーダンス面の軌跡



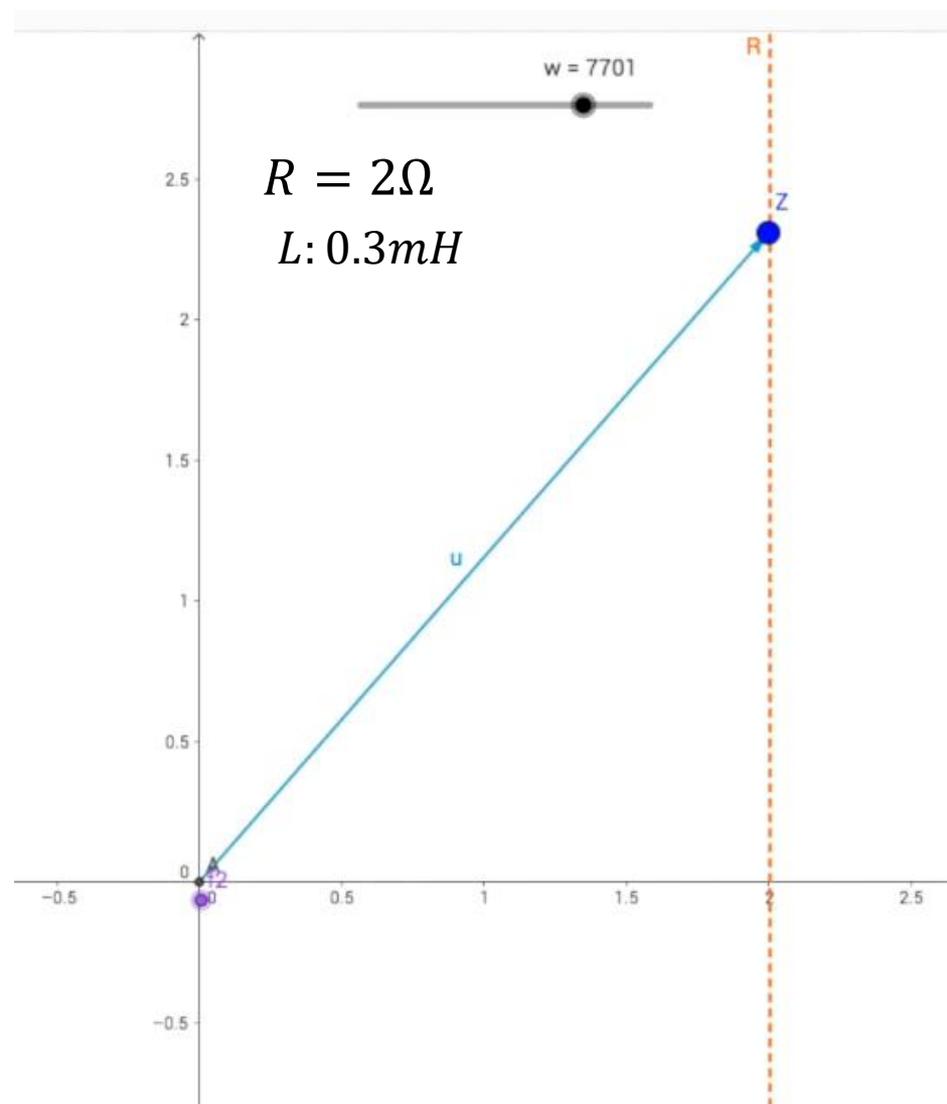
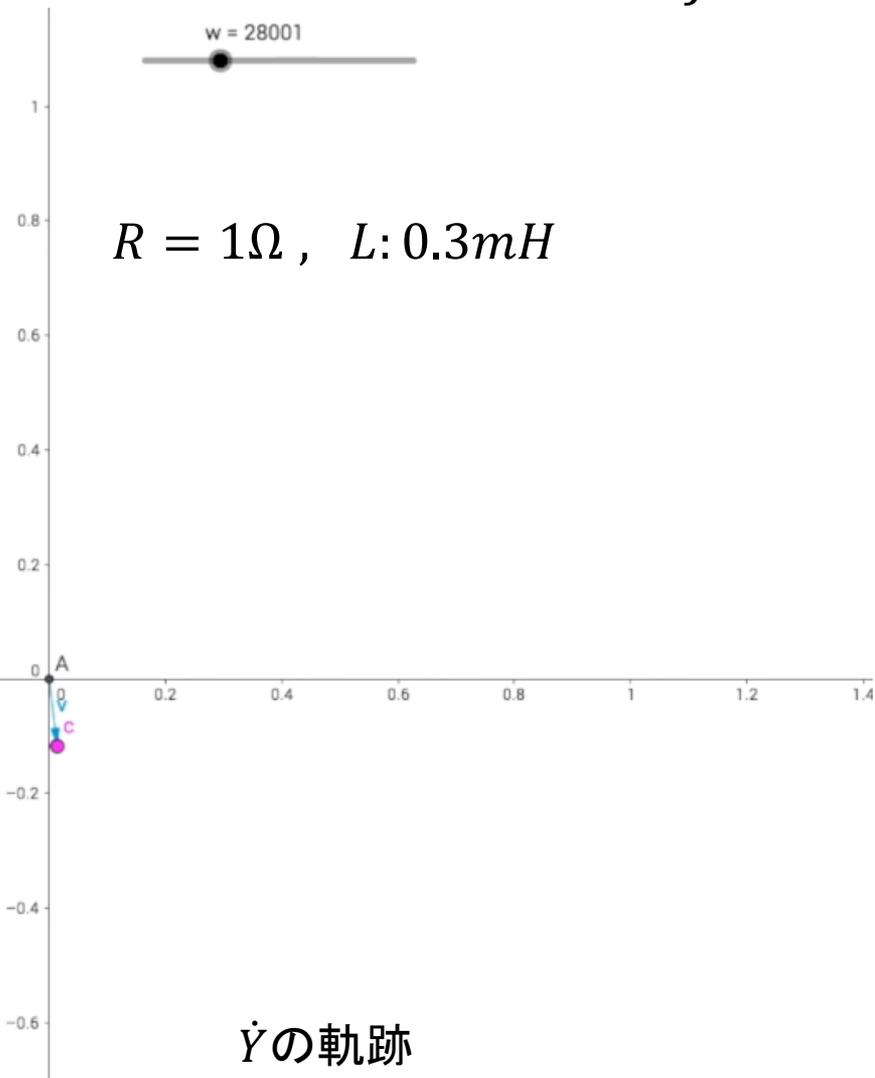
$$\text{インピーダンス: } \dot{Z} = R + j\omega L$$

$$\text{インピーダンス: } \dot{Z} = R + jX$$



# R-L直列回路のアドミタンス面の軌跡

$$\text{アドミタンス: } \dot{Y} = \frac{1}{R + j\omega L}$$



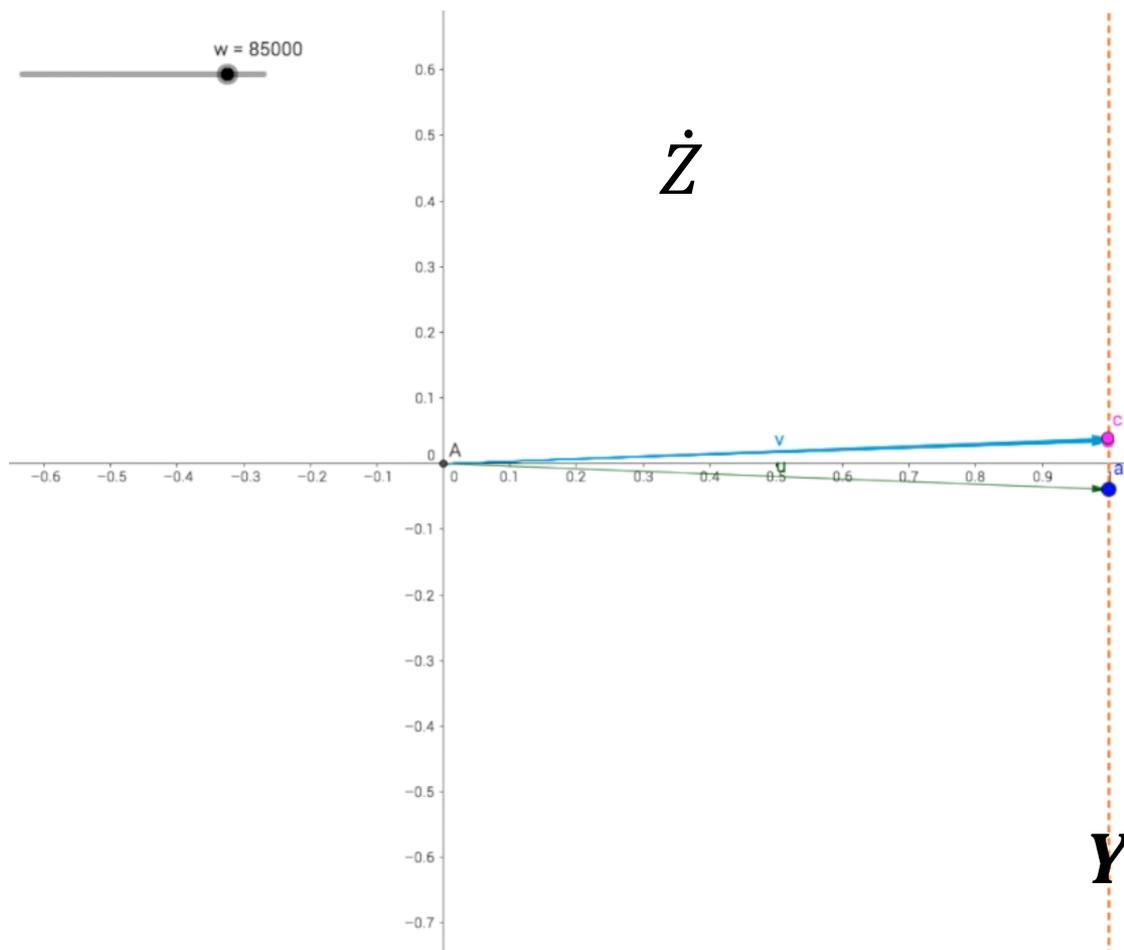
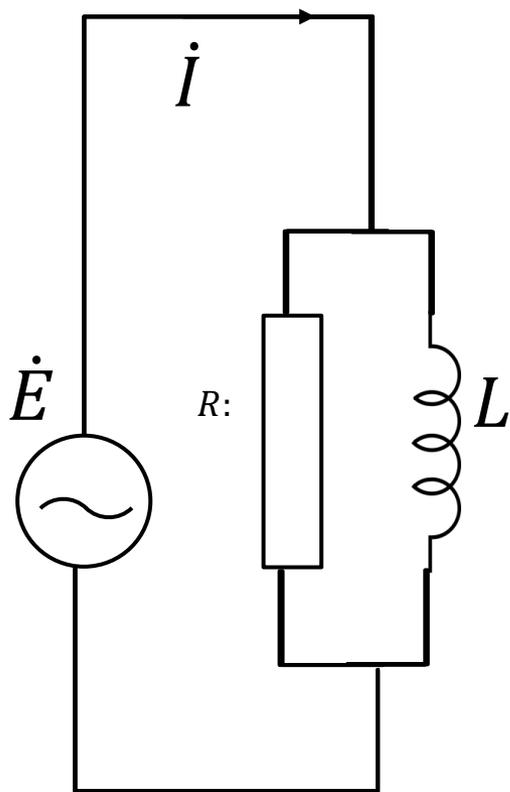
# R-L並列回路のアドミタンス面とインピーダンスの軌跡

$$\dot{Y} = G + jB$$

$$\dot{Y} = \frac{1}{\dot{Z}} = \frac{1}{R} + \frac{1}{j\omega L}$$

$$\dot{Y} = \frac{1}{R} - j\frac{1}{\omega L}$$

$$\dot{Z} = \frac{1}{\dot{Y}}$$



# 究極のフィルタ回路：共振回路

**目的：**

ある特定の周波数だけを  
通過する（通過しない）回路

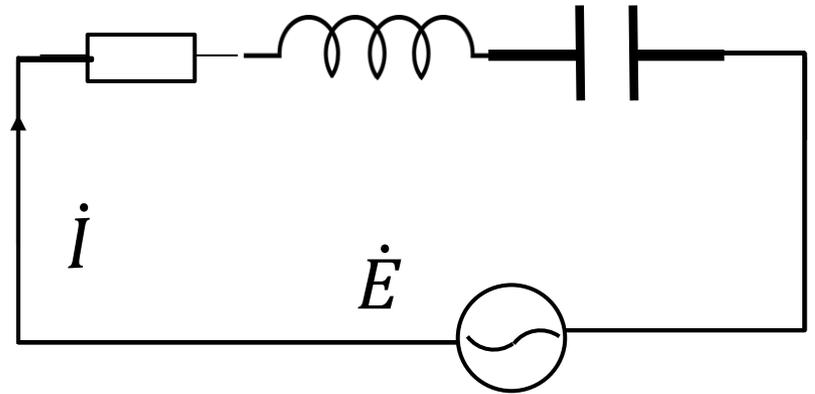
# R-L-C直列共振回路:

$$\dot{Z} = R + j\omega L + \frac{1}{j\omega C}$$

$$= R + j\omega L - j\frac{1}{\omega C}$$

$$= R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$= \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \angle \tan^{-1} \frac{\omega L - \frac{1}{\omega C}}{R} = Z \angle \theta$$



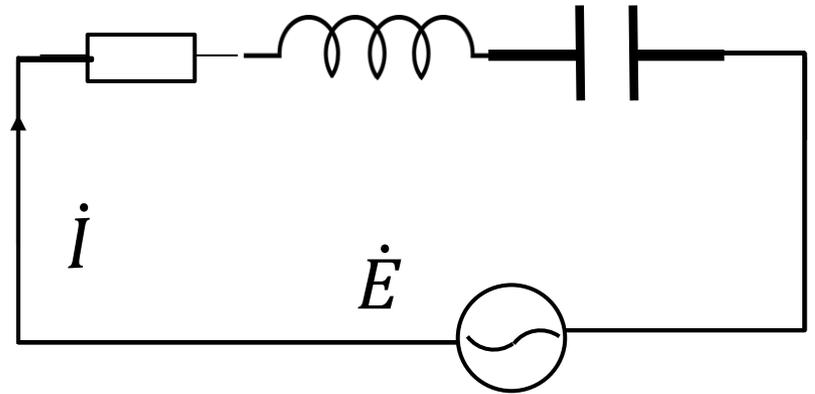
# R-L-C直列共振回路:

$$\dot{Z} = R + j\omega L + \frac{1}{j\omega C}$$

$$= R + j\omega L - j\frac{1}{\omega C}$$

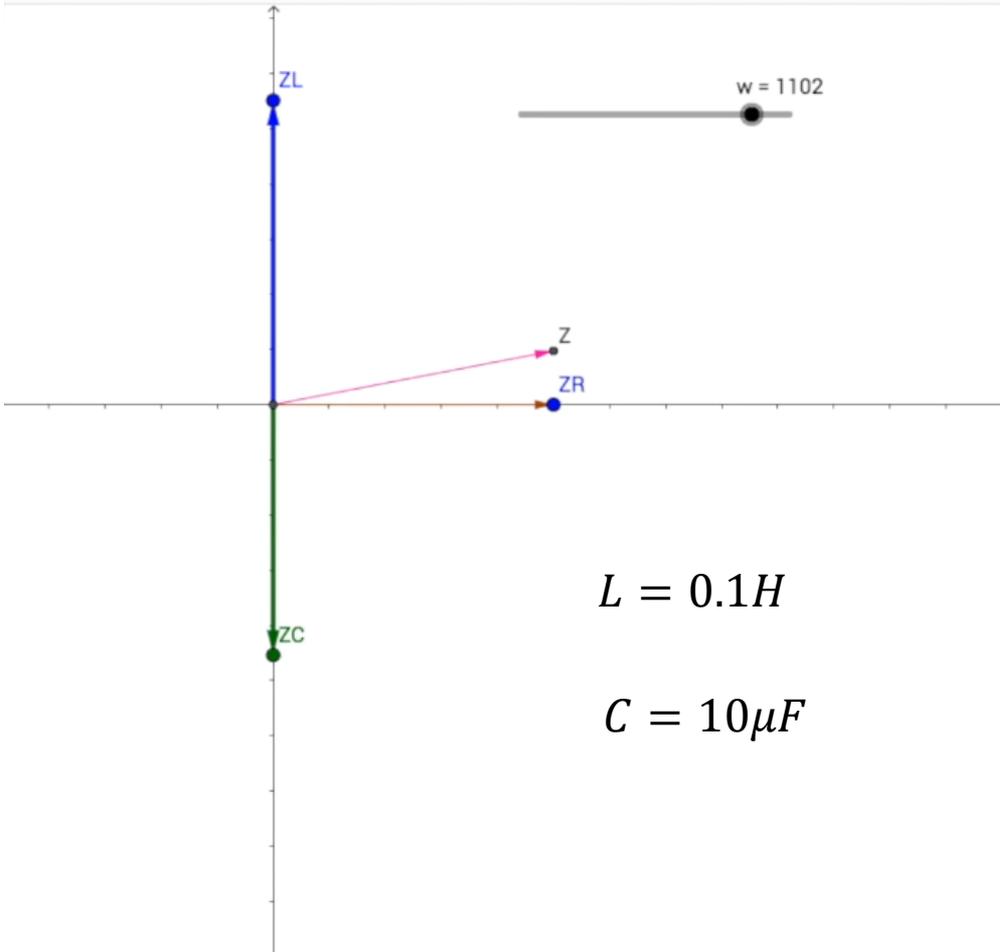
$$= R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$= \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \angle \tan^{-1} \frac{\omega L - \frac{1}{\omega C}}{R} = Z \angle \theta$$



# R-L-C直列共振回路のインピーダンス面の軌跡:

$$\dot{Z} = R + j\omega L + \frac{1}{j\omega C}$$



$$\dot{Z}_L = j\omega L$$

$$\dot{Z}_C = \frac{1}{j\omega C}$$

$$|\dot{Z}| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

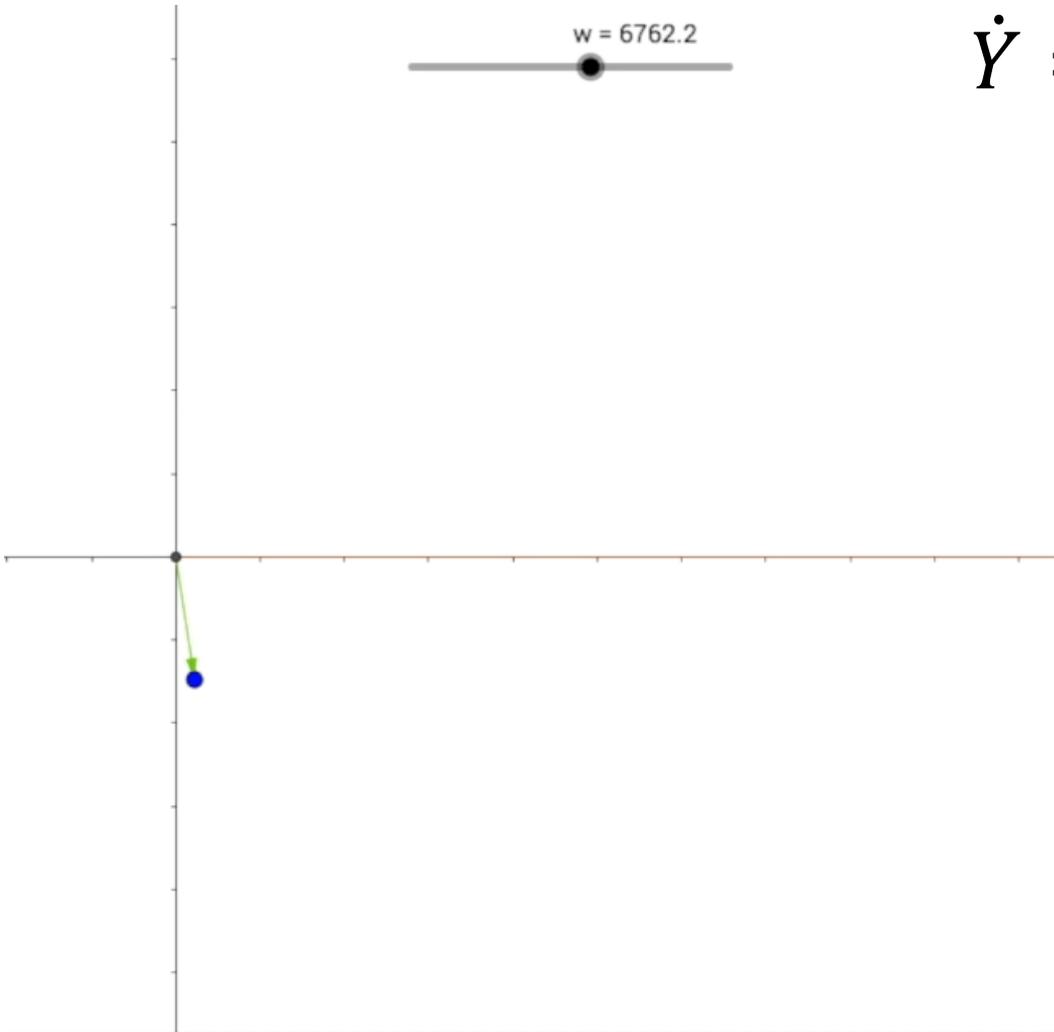
$$|\dot{\mathbf{Z}}_C| = |\dot{\mathbf{Z}}_L|$$

$$|\mathbf{I} * \dot{\mathbf{Z}}_C| = |\mathbf{I} * \dot{\mathbf{Z}}_L|$$

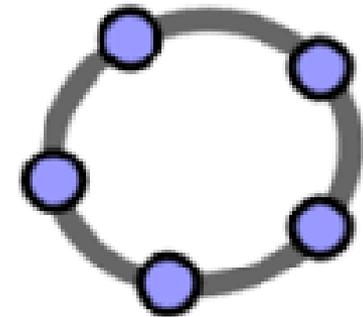
$$|\dot{\mathbf{V}}_C| = |\dot{\mathbf{V}}_L|$$

コイルとコンデンサにかかる電圧の大きさが同じ  
なとき、共振が起きる

# R-L-C直列共振回路のアドミタンス面の軌跡:



$$\dot{Y} = \frac{1}{\dot{Z}} = \frac{1}{R + j\omega L + \frac{1}{j\omega C}}$$



GeoGebra

# R-L-C直列共振回路：電流の周波数依存性：

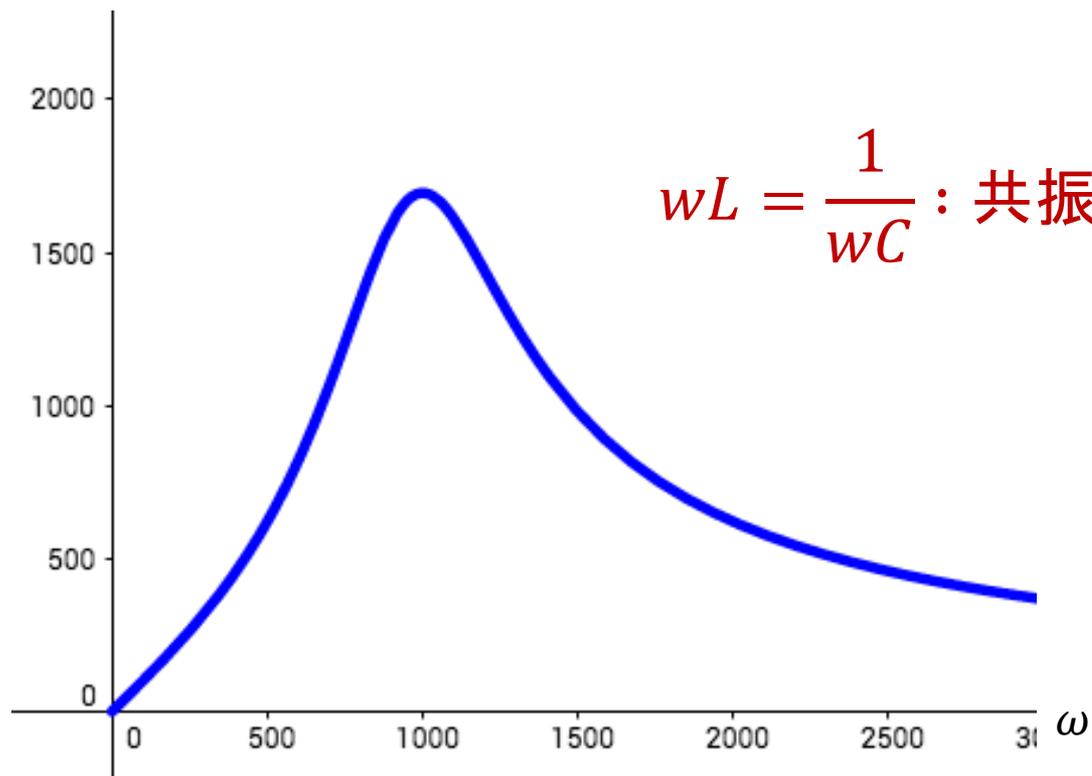
$$\omega L - \frac{1}{\omega C} = 0$$

$$\omega_0^2 = \frac{1}{LC}$$

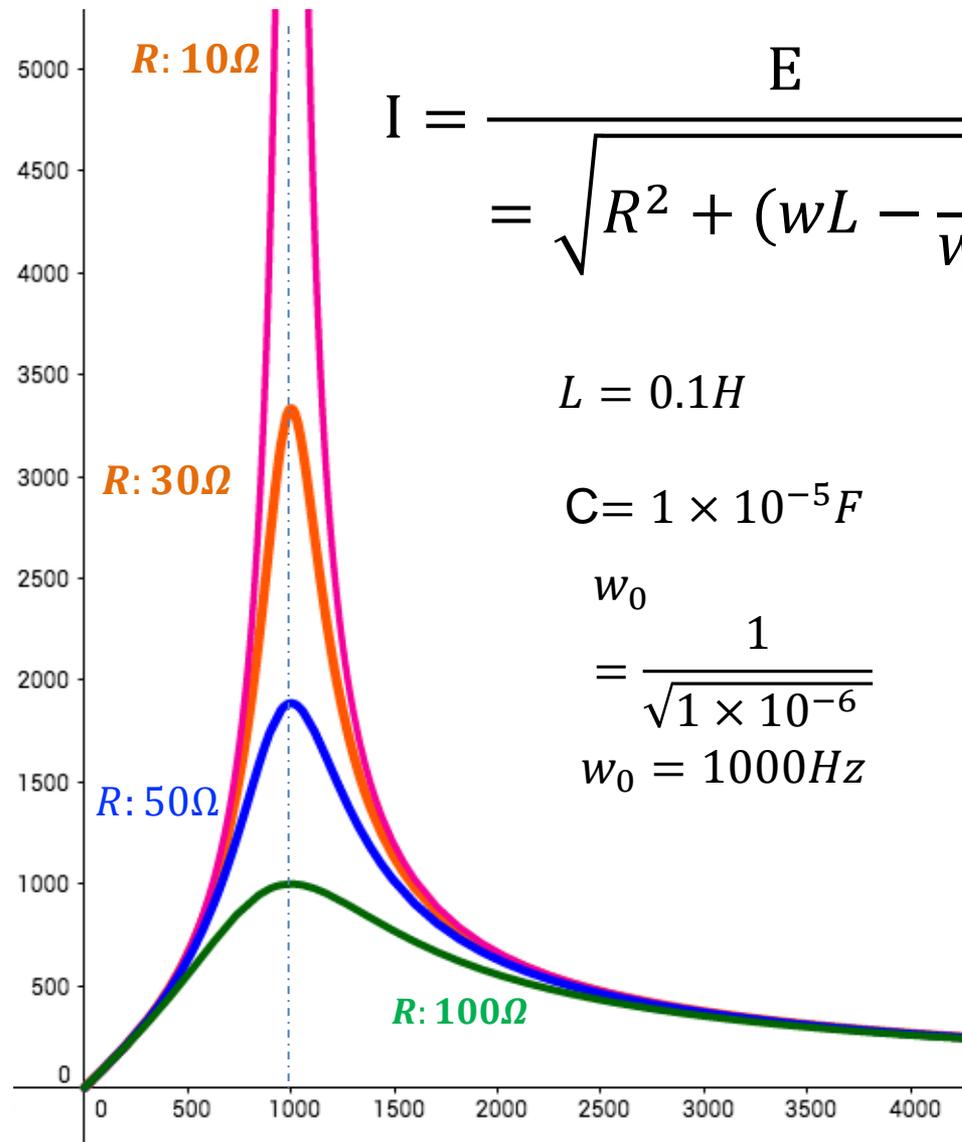
$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$i = \frac{1}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} E$$



# 共振の条件:



$$I = \frac{E}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$

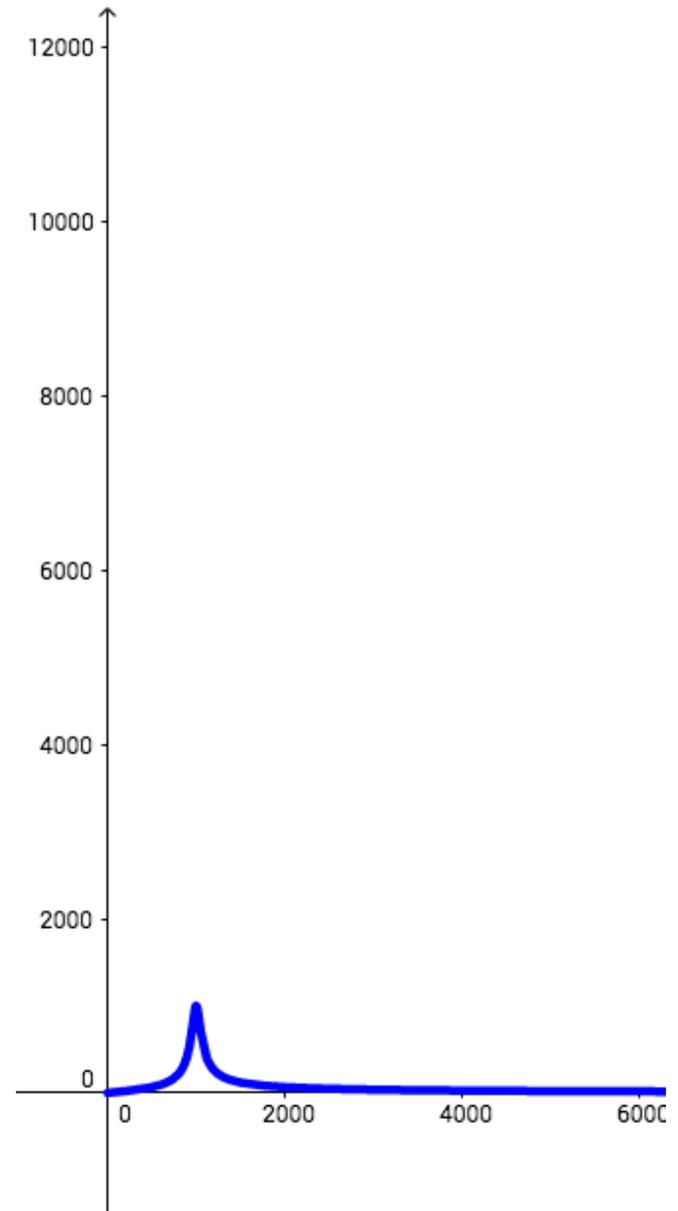
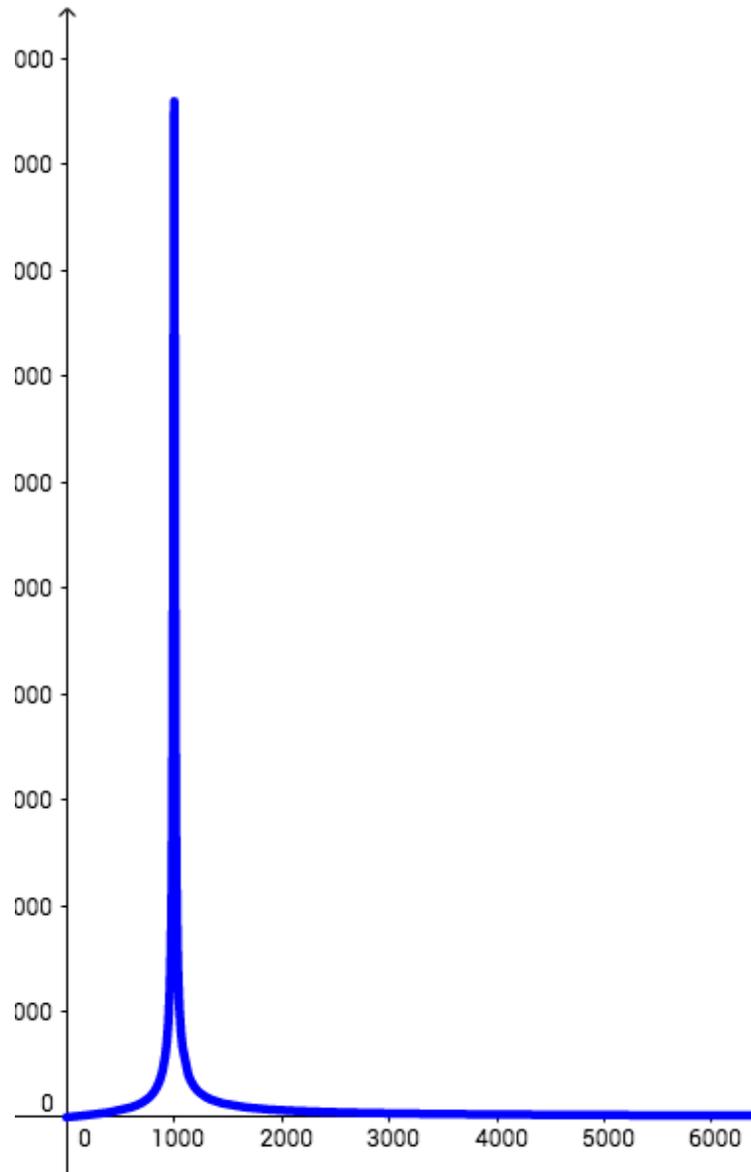
$$L = 0.1H$$

$$C = 1 \times 10^{-5}F$$

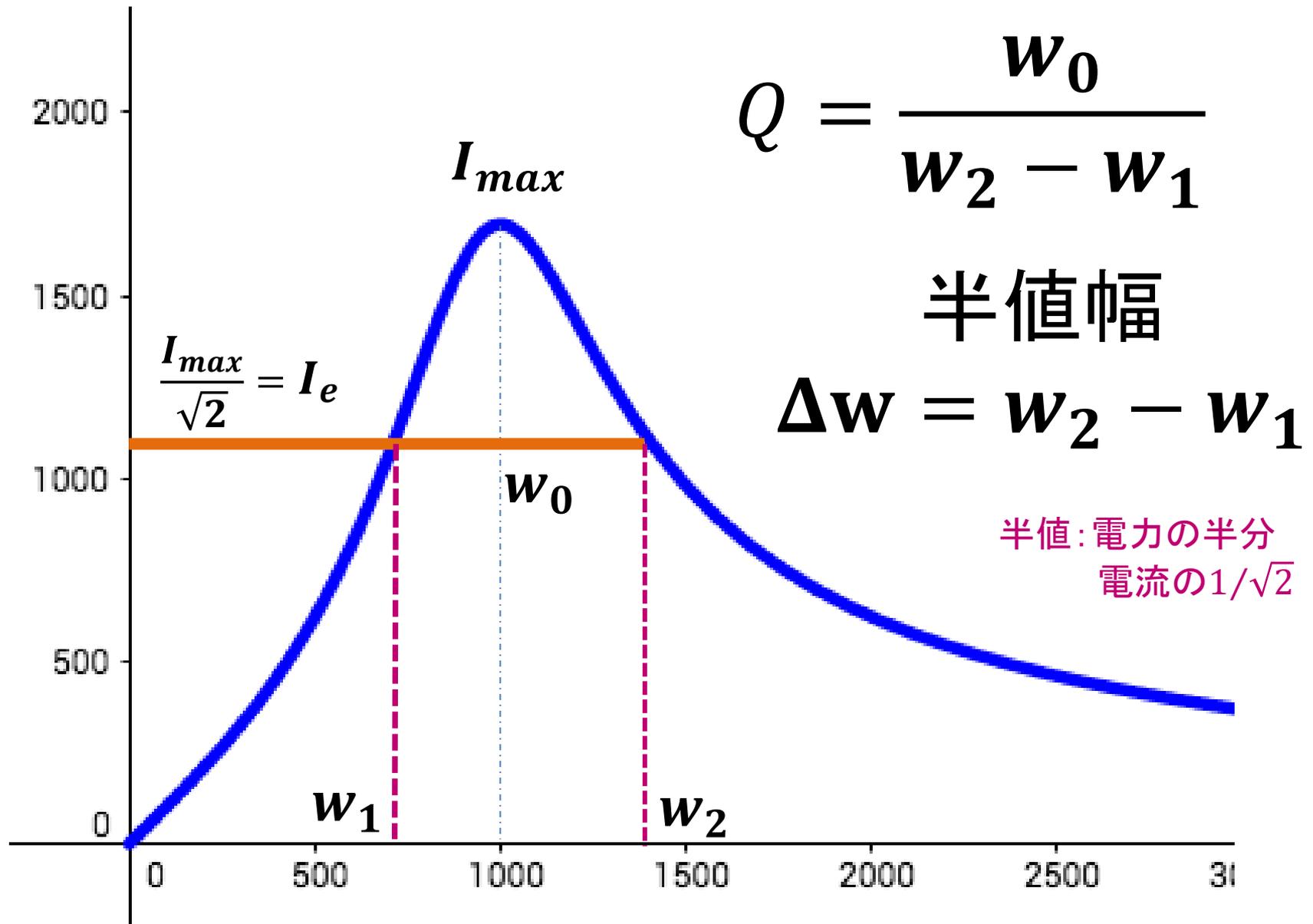
$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\omega_0 = 1000Hz$$

# 共振ピークの鋭さ:



# 共振ピークの鋭さを表すQ値:



# R-L-C直列共振回路におけるQ値の計算式を導く

$$I = \frac{|\dot{E}|}{|\dot{Z}|} = \frac{E}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$I = \frac{|\dot{E}|}{|\dot{Z}|} = \frac{E}{R} \frac{1}{\sqrt{1 + \frac{\left(\omega L - \frac{1}{\omega C}\right)^2}{R^2}}}$$

$$I = \frac{|\dot{E}|}{|\dot{Z}|} = I_{max} \frac{1}{\sqrt{1 + \frac{\left(\omega L - \frac{1}{\omega C}\right)^2}{R^2}}}$$

## R-L-C直列共振回路におけるQ値の計算式を導く

$$\frac{(wL - \frac{1}{wC})^2}{R^2} = 1 \qquad \frac{wL - \frac{1}{wC}}{R} = \pm 1$$

$$w^2 - w \frac{R}{L} - \frac{1}{LC} = 0$$

$$w_2 = \frac{1}{2} \left( \frac{R}{L} + \sqrt{\left(\frac{R}{L}\right)^2 + \frac{4}{LC}} \right)$$

## R-L-C直列共振回路におけるQ値の計算式を導く

$$\omega^2 + \omega \frac{R}{L} - \frac{1}{LC} = 0$$

$$\omega_1 = \frac{1}{2} \left( -\frac{R}{L} + \sqrt{\left(\frac{R}{L}\right)^2 + \frac{4}{LC}} \right)$$

$$Q = \frac{\omega_0}{\omega_2 - \omega_1} = \frac{\omega_0 L}{R} = \frac{I * |j\omega_0 L|}{I * R}$$

$$\begin{aligned} Q &= \frac{\omega_0}{\omega_2 - \omega_1} = \frac{\omega_0 L}{R} \\ &= \frac{I^* |j\omega_0 L|}{I^* R} \\ &= \frac{I^* |j\omega_0 L|}{I^* R} = \frac{|I^* j\omega_0 L|}{|I^* R|} = \frac{|V_L|}{|V_R|} \end{aligned}$$

コイルにかかる電圧と抵抗にかかる電圧の大きさの比である

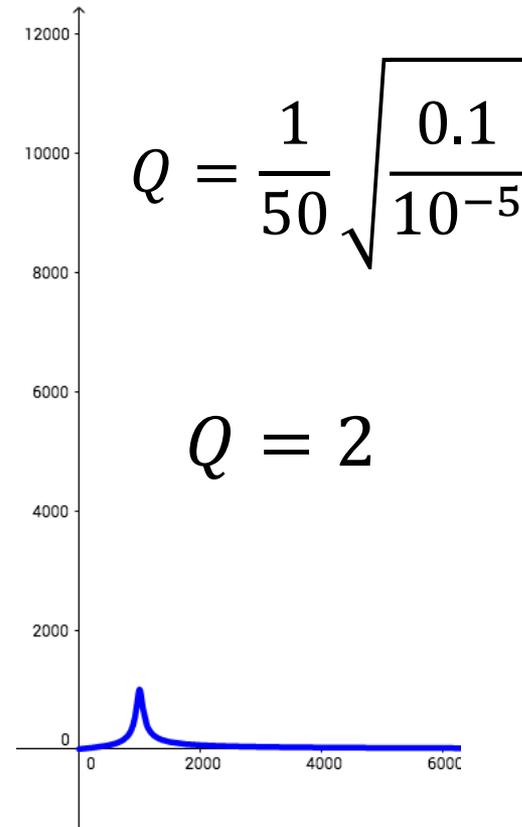
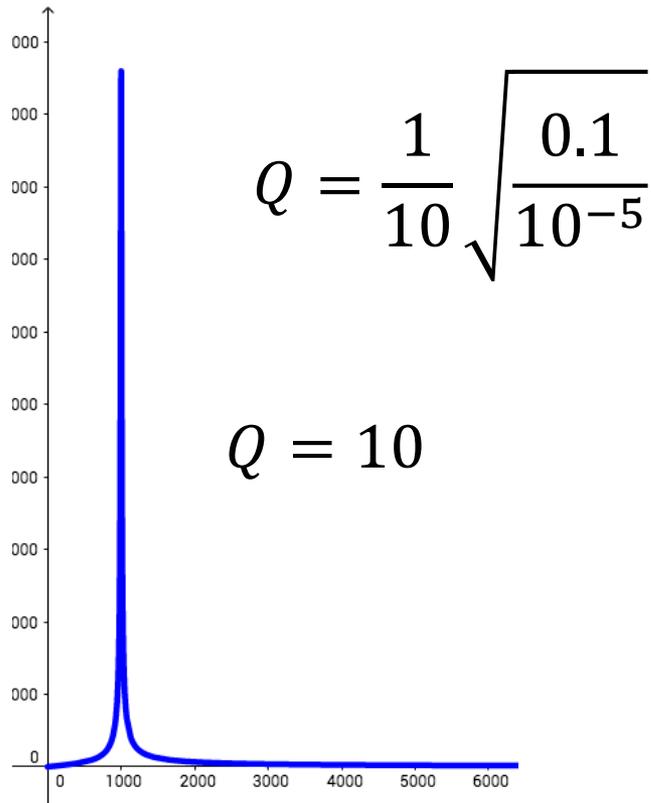
$$\omega_0^2 = \frac{1}{LC} \quad Q = \frac{\omega_0 L}{R} = \frac{\omega_0 \frac{1}{\omega_0^2 C}}{R} = \frac{1}{\omega_0 C R}$$

$$Q = \frac{\omega_0 L}{R} = \frac{\omega_0 \frac{1}{\omega_0^2 C}}{R} = \frac{\left| I^* \frac{1}{j\omega_0 C} \right|}{|I^* R|} = \frac{|V_C|}{|V_R|}$$

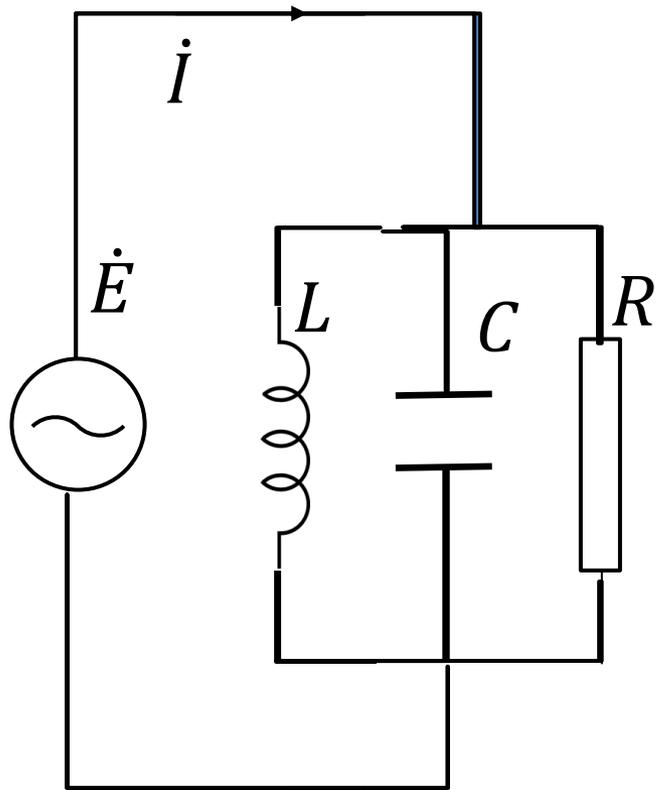
# R-L-C直列共振回路におけるQ値を試してみる！

$$Q = \frac{L}{R} \sqrt{\frac{1}{LC}}$$

$$Q = \frac{1}{R} \sqrt{\frac{L}{C}}$$



# R-L-C 並列共振回路：



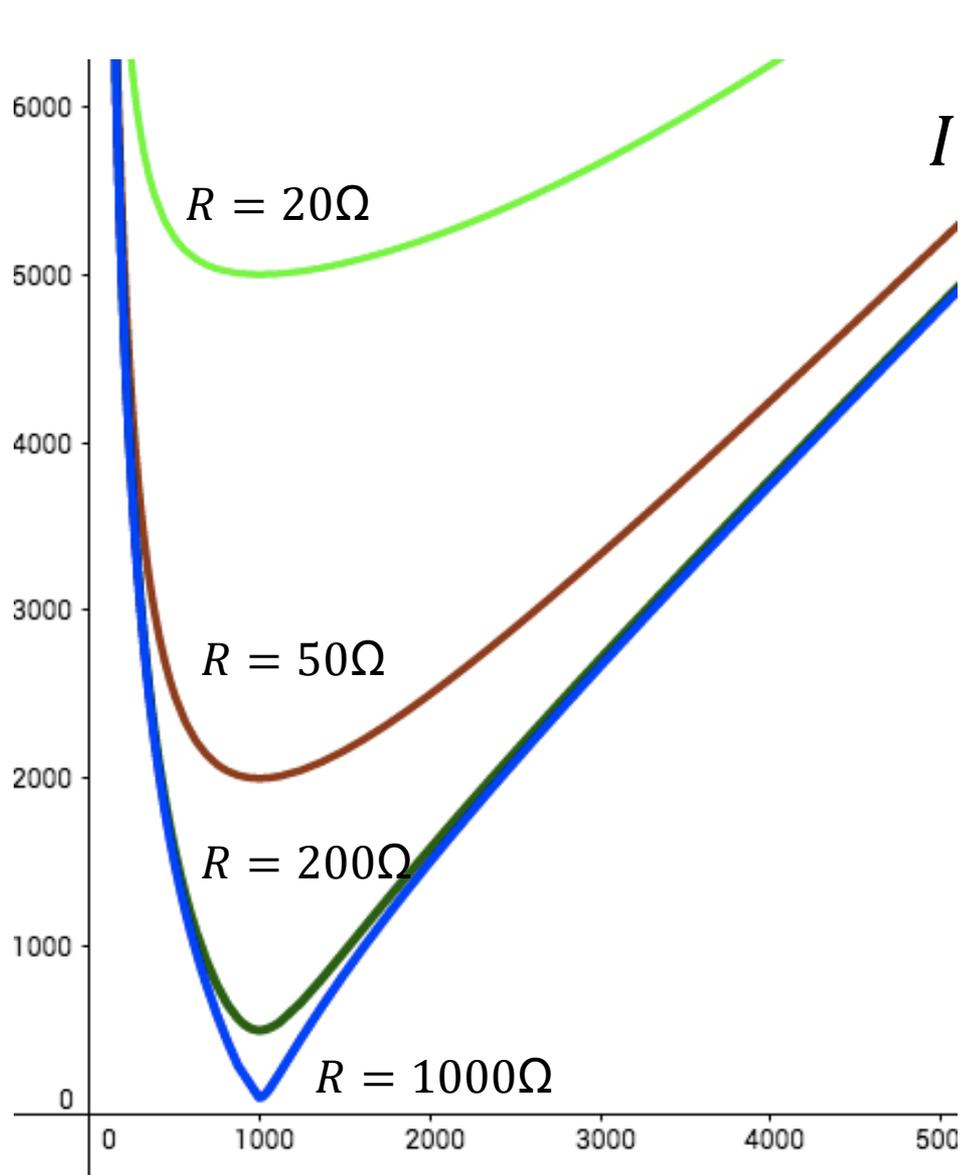
$$\dot{Y} = \frac{1}{\dot{Z}} = \frac{1}{R} + \frac{1}{j\omega L} + j\omega C$$

$$\dot{I} = \dot{Y}\dot{E} = \left(\frac{1}{R} + \frac{1}{j\omega L} + j\omega C\right) \dot{E}$$

$$\dot{I} = \dot{Y}\dot{E} = \left\{\frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)\right\} \dot{E}$$

$$I = YE = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\omega C - \frac{1}{\omega L}\right)^2} E$$

# R-L-C並列共振回路における電流の周波数依存性



$$I = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\omega C - \frac{1}{\omega L}\right)^2} E$$

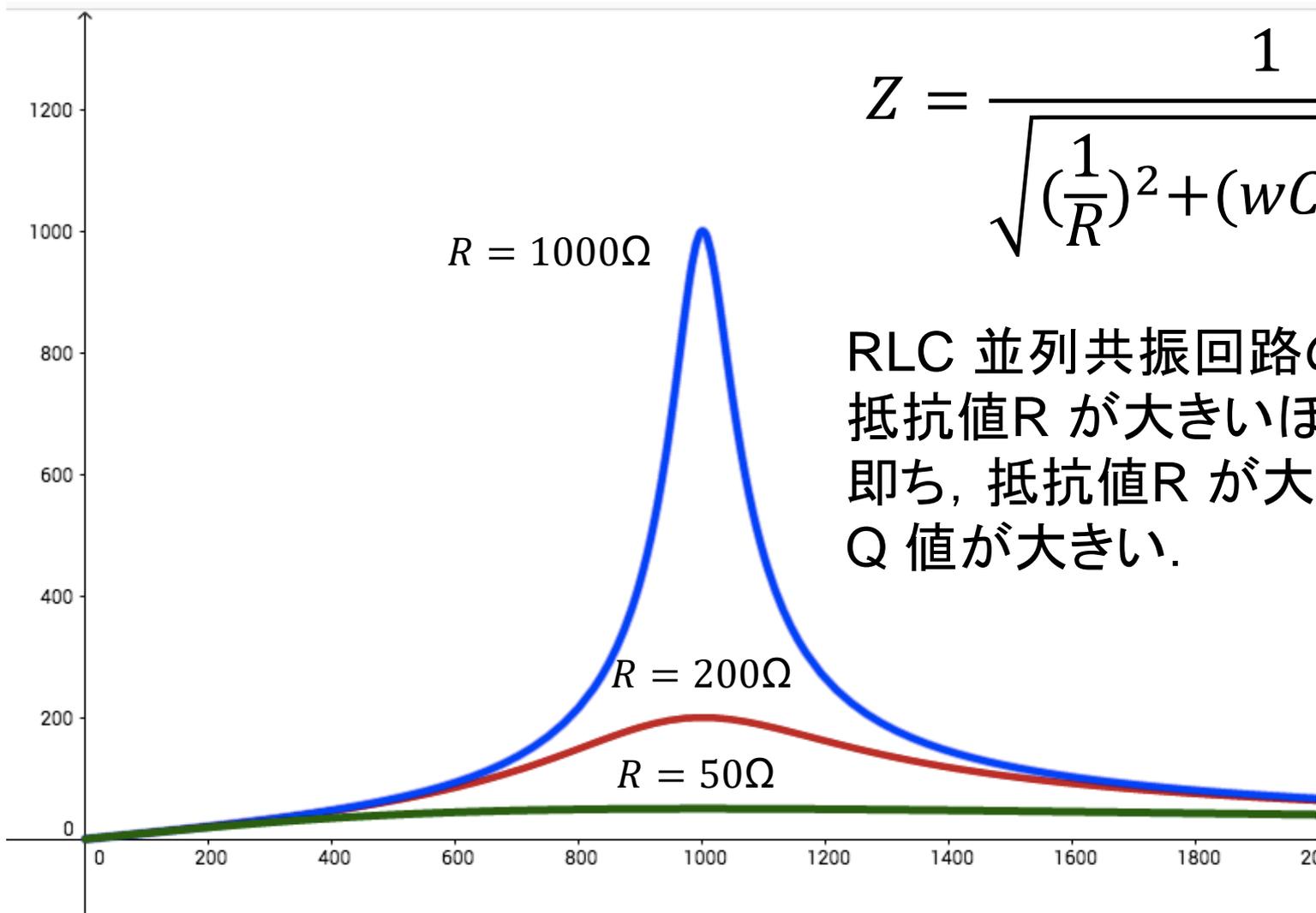
$$L = 0.1H$$

$$C = 1 \times 10^{-5}F$$

$$\omega_0 = \frac{1}{\sqrt{1 \times 10^{-6}}}$$

$$\omega_0 = 1000Hz$$

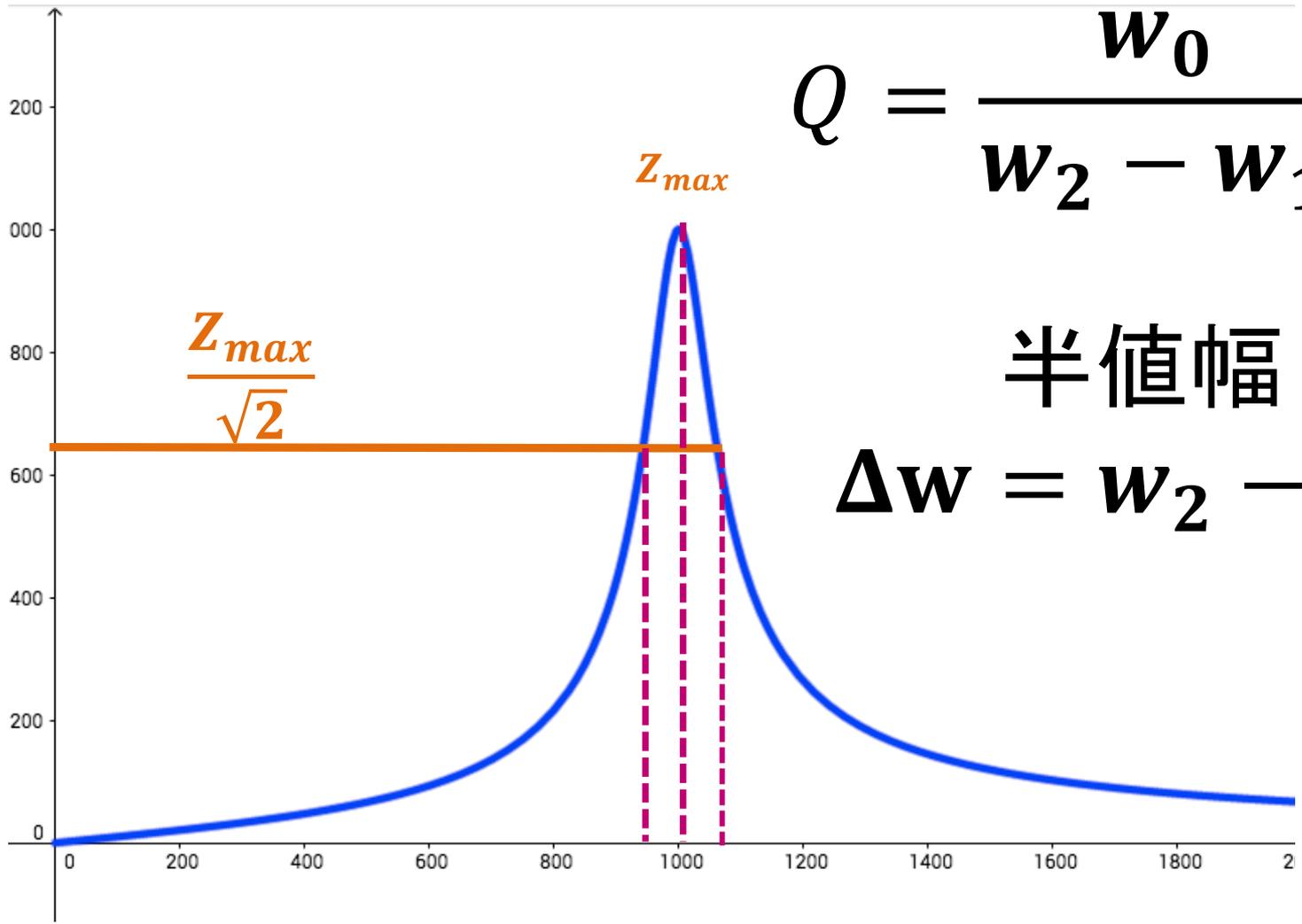
# R-L-C並列共振回路のインピーダンスの周波数依存性



$$Z = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\omega C - \frac{1}{\omega L}\right)^2}}$$

RLC 並列共振回路の鋭さは、抵抗値  $R$  が大きいほど鋭い。即ち、抵抗値  $R$  が大きいほど  $Q$  値が大きい。

# 並列共振回路のQ値：インピーダンスを使う



$$Q = \frac{\omega_0}{\omega_2 - \omega_1}$$

半値幅

$$\Delta\omega = \omega_2 - \omega_1$$

## R-L-C並列共振回路におけるQ値の計算式を導く

$$Z = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\omega C - \frac{1}{\omega L}\right)^2}}$$

$$\omega C - \frac{1}{\omega L} = \pm \frac{1}{R}$$

$$\left\{ \begin{array}{l} \omega^2 - \omega \frac{1}{RC} - \frac{1}{LC} = 0 \\ \omega^2 + \omega \frac{1}{RC} - \frac{1}{LC} = 0 \end{array} \right.$$

## R-L-C並列共振回路におけるQ値の計算式を導く

$$\omega_2 = \frac{1}{2LCR} \left\{ L + \sqrt{L^2 + 4LCR^2} \right\}$$

$$\omega_1 = \frac{1}{2LCR} \left\{ -L + \sqrt{L^2 + 4LCR^2} \right\}$$

$$Q = \frac{\omega_0}{\omega_2 - \omega_1} = \omega_0 CR = R \sqrt{\frac{C}{L}}$$

$$Q = \frac{\omega_0}{\omega_2 - \omega_1} = \omega_0 CR = R \sqrt{\frac{C}{L}}$$

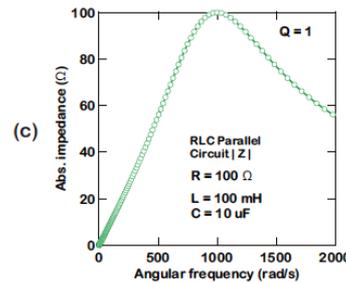
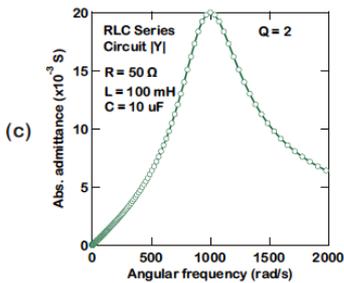
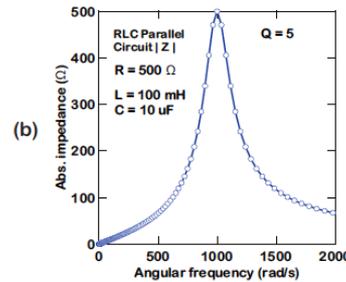
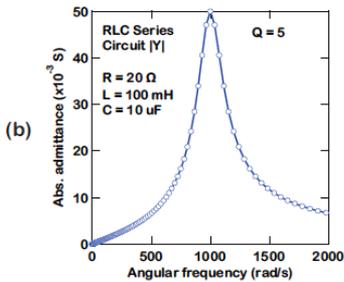
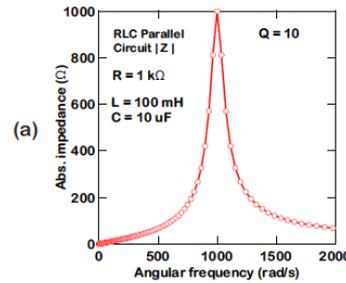
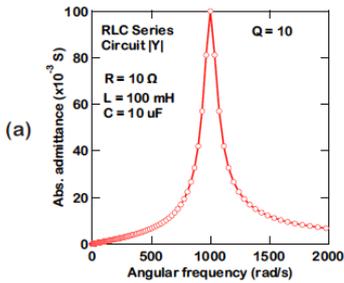
$$Q = \omega_0 CR = \frac{\omega_0 C}{\frac{1}{R}} = \frac{V * \omega_0 C}{V * \frac{1}{R}}$$

$$= \frac{V / \left(\frac{1}{\omega_0 C}\right)}{V * \frac{1}{R}} = \frac{V / \left(\frac{1}{|j\omega_0 C|}\right)}{V * \frac{1}{R}}$$

$$\frac{V / \left( \frac{1}{|j\omega_0 C|} \right)}{V * \frac{1}{R}} = \frac{|V/Z_c|}{|V/R|} = \frac{|I_c|}{|I_R|}$$

コンデンサに流れる電流と抵抗に流れる電流の大きさの比である

# 共振回路における抵抗の役割を吟味しましょう

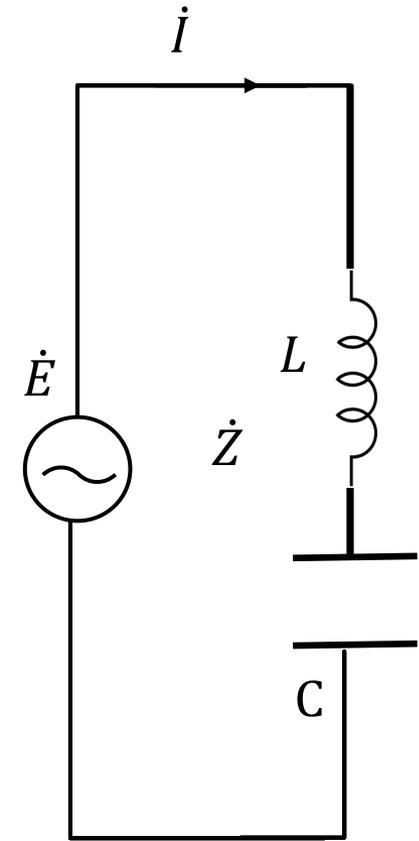


直列共振回路では、R は小さい方がよい(究極の状態は直列接続された抵抗が無い状態:  $R = 0 \Omega$ )・並列共振回路では、R が大きい方がよい(究極の状態は並列接続された抵抗が無い状態:  $R = \infty \Omega$ )  
ということは、

最初から抵抗なんか接続せずに、LC 直列共振回路、LC 並列共振回路にすればよいのである。そうすれば共振特性の鋭さを表すQ値は無限大となり、究極の鋭さを持つ回路を作ることができる\*3。

なぜ、わざわざ抵抗の入った回路について勉強したのか？

# 純粋なLC共振回路



$$\dot{Z} = j\omega L + \frac{1}{j\omega C}$$

$$\dot{Z} = j\omega L - j\frac{1}{\omega C}$$

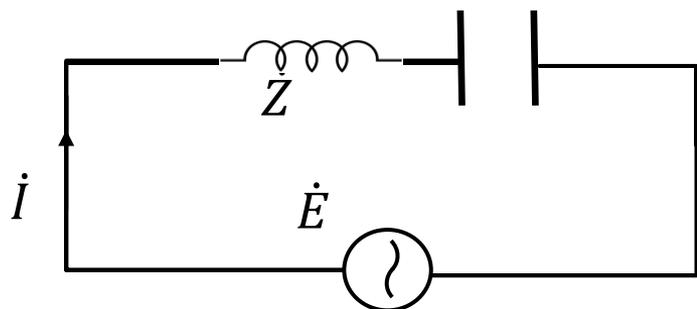
$$\dot{Z} = j\left(\omega L - \frac{1}{\omega C}\right)$$

$$\dot{Z} = \left(\omega L - \frac{1}{\omega C}\right) \angle \theta$$

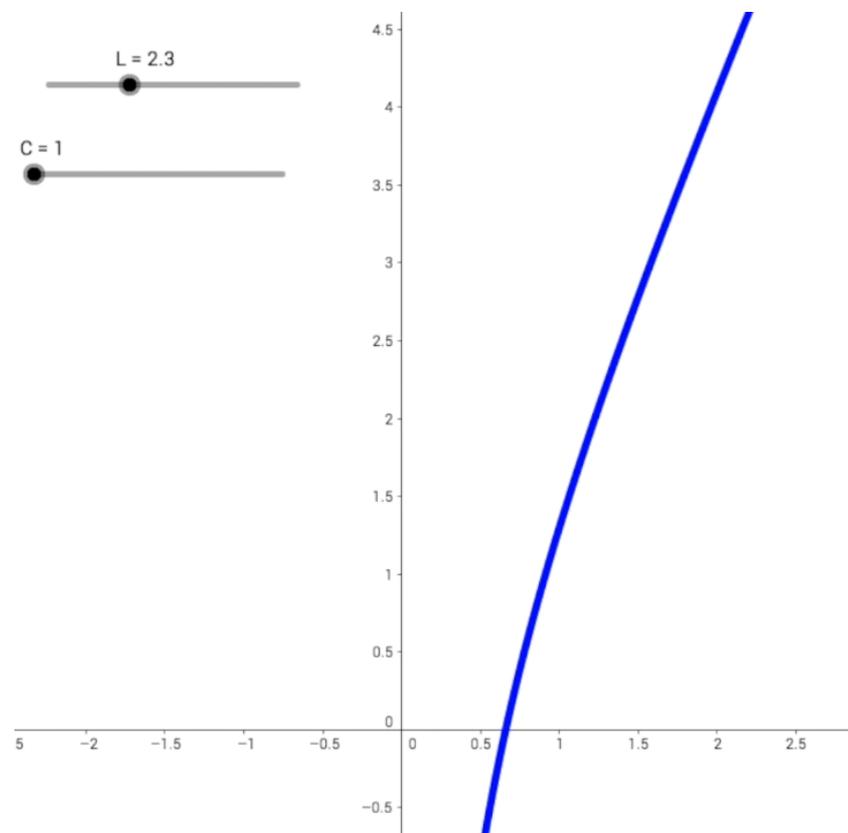
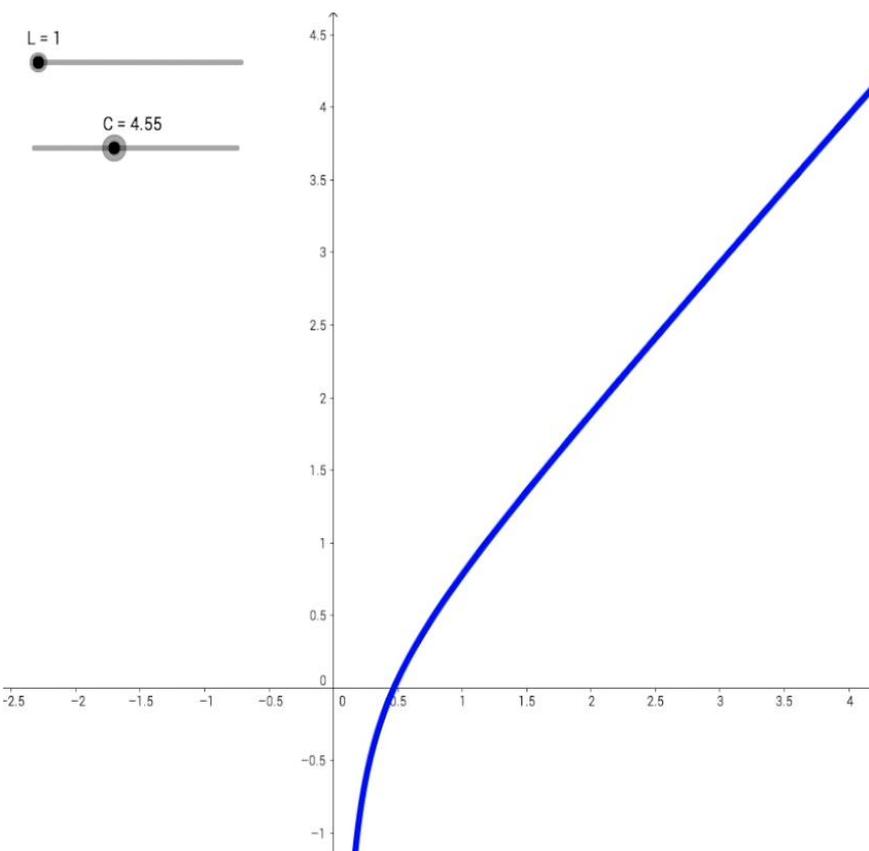
$$\theta = 90, \omega L > \frac{1}{\omega C}$$

$$\theta = -90, \omega L < \frac{1}{\omega C}$$

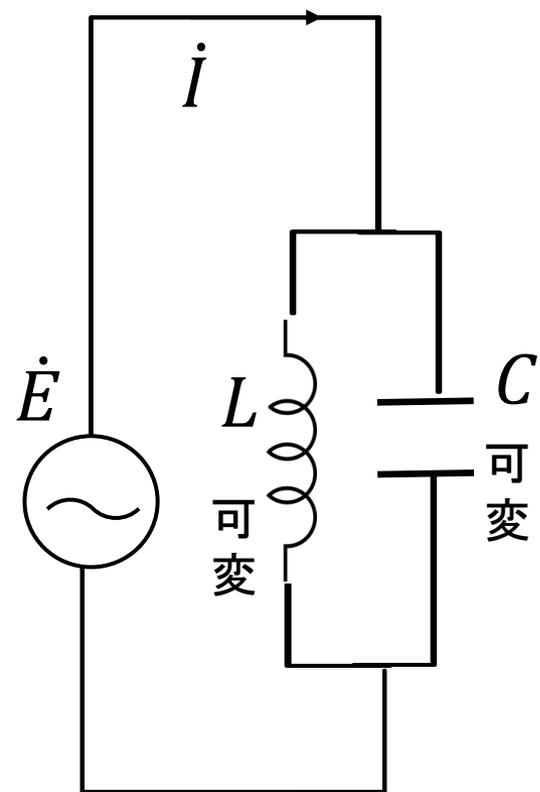
# 純粋なLC共振回路



$$\dot{Z} = \left( \omega L - \frac{1}{\omega C} \right) \angle \theta$$

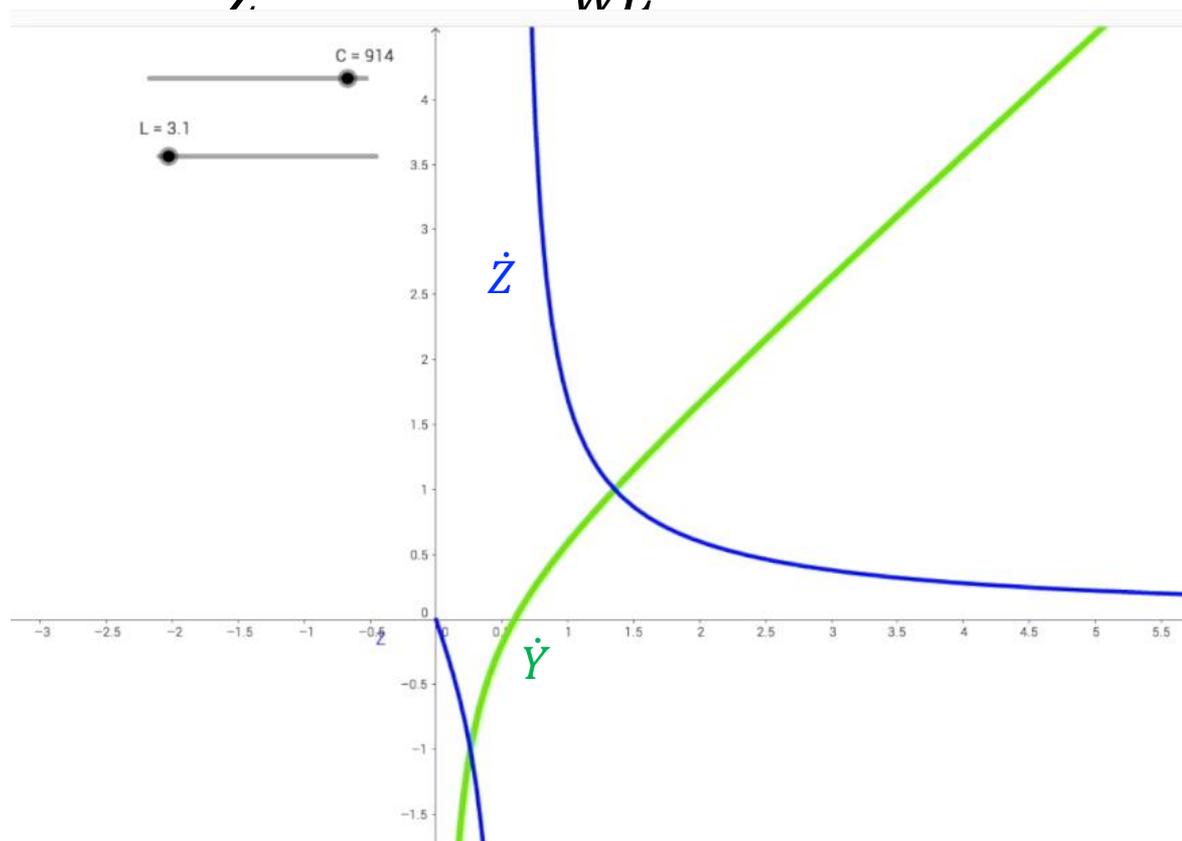


# 純粋なLC共振回路



$$\dot{Y} = \frac{1}{\dot{Z}} = \frac{1}{j\omega L} + j\omega C$$

$$\dot{Y} = \frac{1}{\dot{Z}} = j\omega C - j\frac{1}{\omega L}$$



# 抵抗のないコイルやコンデンサは存在しない！

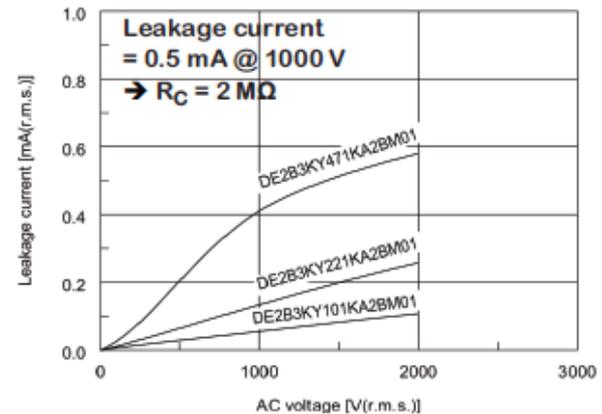
## 電気的特性

インダクタンス (μH)	インダクタンス 許容差	直流抵抗 (Ω)max.
10	±10%	0.019
15	±10%	0.022
22	±10%	0.031
33	±10%	0.044
47	±10%	0.059
68	±10%	0.073
100	±10%	0.1
150	±10%	0.15
220	±10%	0.26
330	±10%	0.32
470	±10%	0.48
680	±10%	0.73
1000	±10%	0.96
1500	±10%	1.4
2200	±10%	2.5
3300	±10%	3.3
5600	±10%	6.4

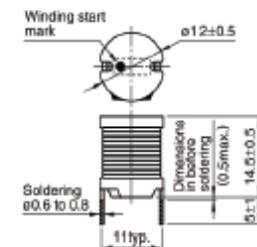
品番	交流定格電圧 (Vac)	温度特性	静電容量 (pF)
DEJE3E2102Z□□□	250	E	1000 +80/-20%
DEJE3E2222Z□□□	250	E	2200 +80/-20%
DEJE3E2332Z□□□	250	E	3300 +80/-20%
DEJE3E2472Z□□□	250	E	4700 +80/-20%
DEJF3E2472Z□□□	250	F	4700 +80/-20%
DEJF3E2103Z□□□	250	F	10000 +80/-20%

Type KY (B char.)

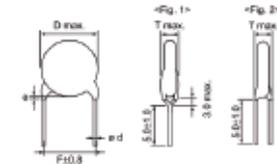
AC voltage : 60Hz  
Temperature : 25°C



形状・寸法  
共線タイプ (10 ~ 100μH)



**muRata**

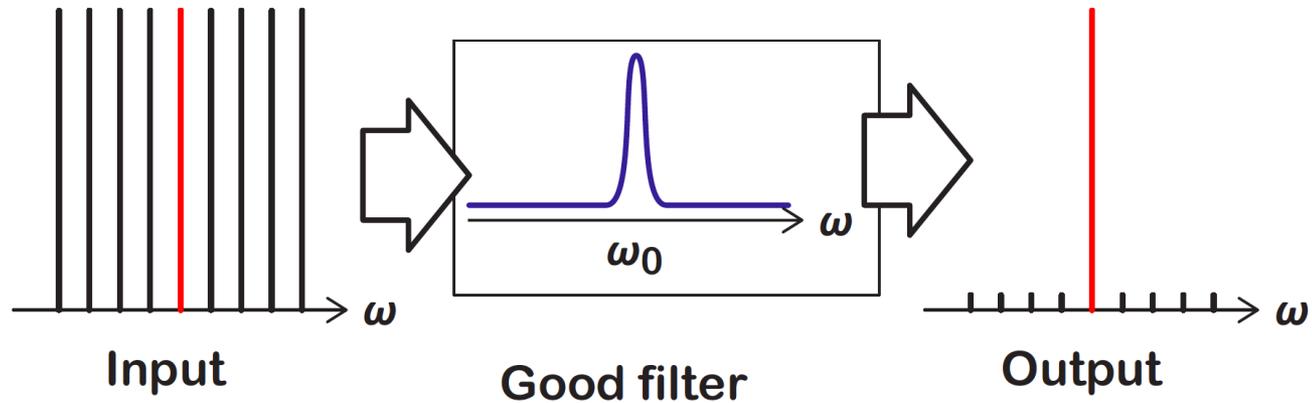


(in mm)

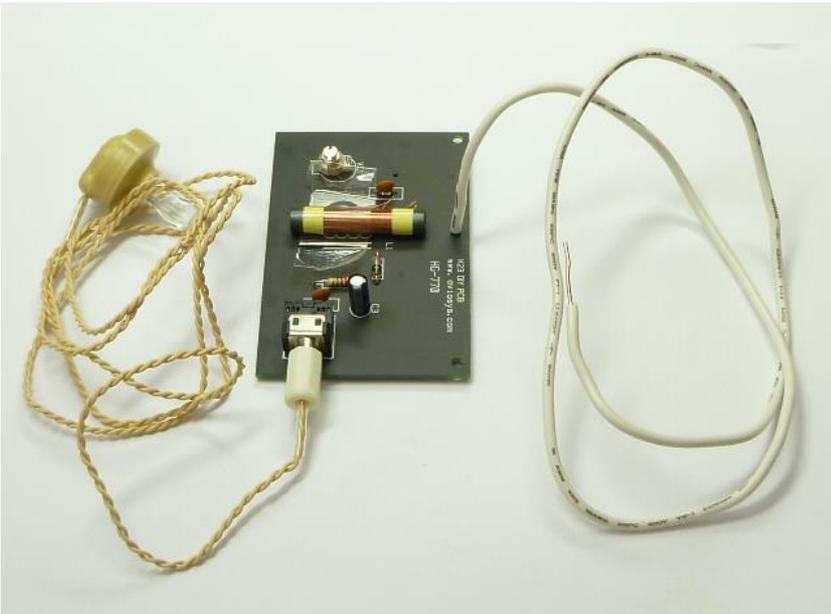
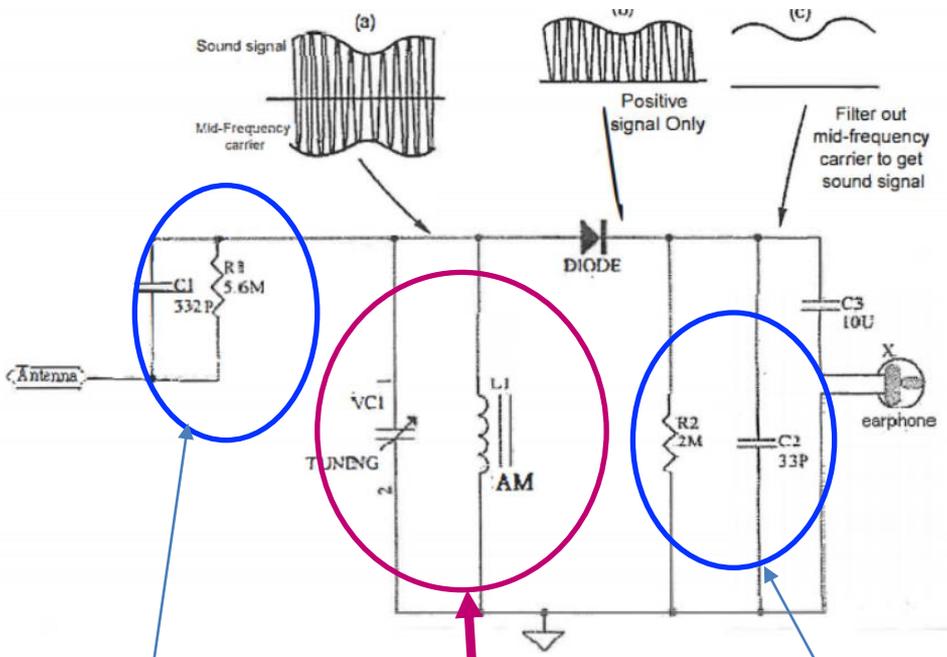
Lead Code	Coating Extension	ø	ø d	Style
B3	Up to the end of crimp	0.8±0.05	0.8±0.05	Fig. 1
D3	3.0 max.	0.8±0.05	0.8±0.05	Fig. 2

〔部品〕  
タテクリンブショート(B3)  
ストレートショート(D3)

# 周波数選別に用いられるフィルタの機能



# ゲルマラジオに使用した共振回路

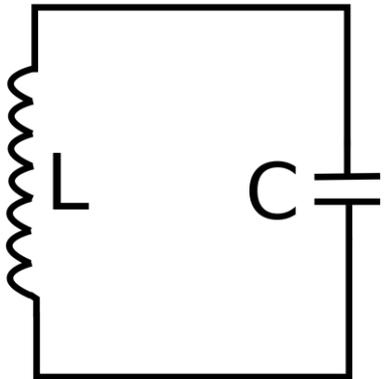


共振回路

ハイパスフィルタ

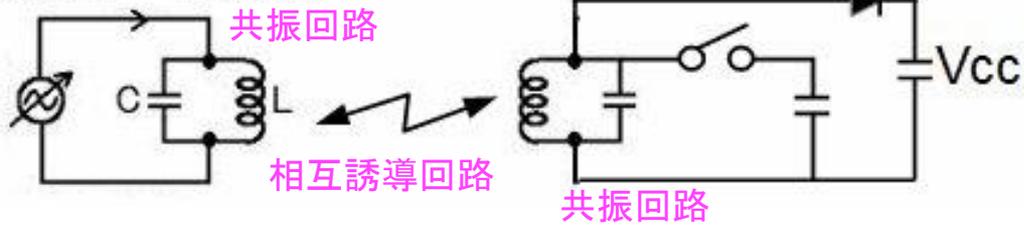
ローパスフィルタ

# 更に、私達の身の回りの共振回路:

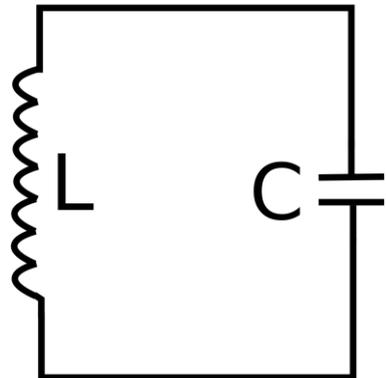
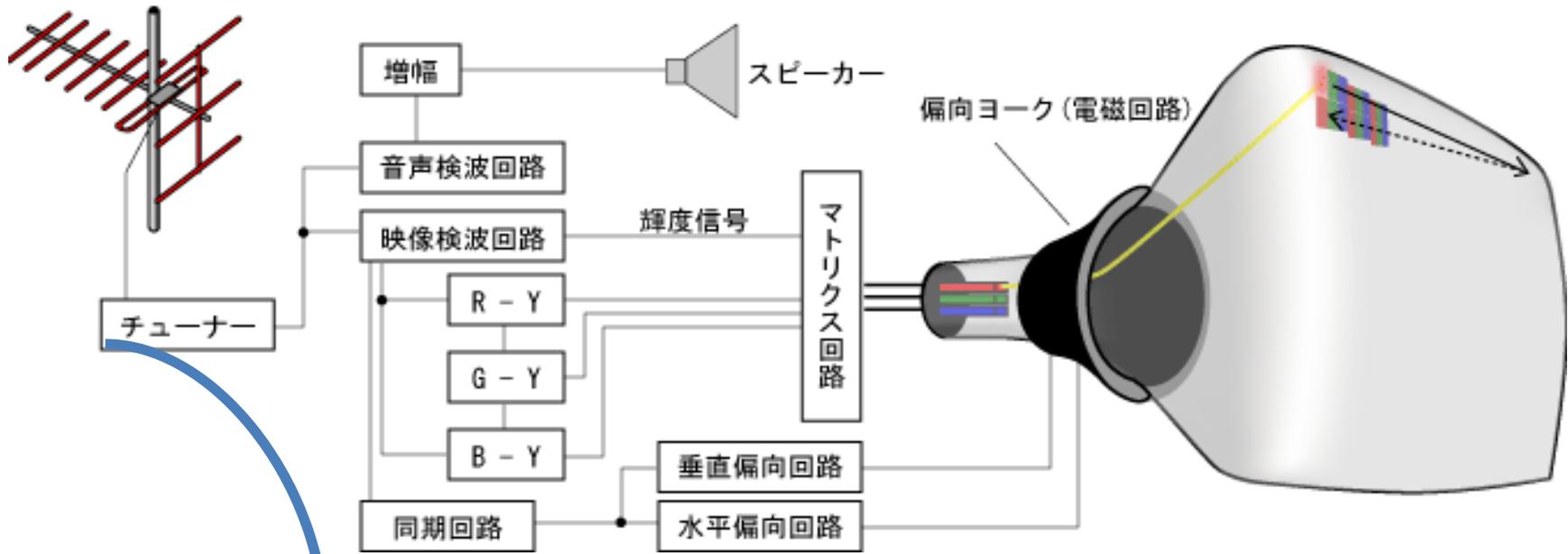


親機 = 改札機

カード = SUICA



# 更に、私達の身の回りの共振回路：



## 共振という条件に注意してください

$$\dot{Z} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \angle \tan^{-1} \frac{\omega L - \frac{1}{\omega C}}{R} = Z \angle \theta$$

$$\dot{I} = \frac{\dot{V}}{\dot{Z}}$$

$$\dot{Z}_L = j\omega L \quad \dot{V}_L = j\omega L * \dot{I}$$

$$\dot{Z}_C = \frac{1}{j\omega C} \quad \dot{V}_C = \frac{1}{j\omega C} * \dot{I}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$Q = \frac{1}{R} \sqrt{\frac{L}{C}}$$

$$Q = \frac{\omega_0 L}{R}$$