



i-PERC

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概要:

□ 復習

□ キャリア密度 $\int D(E)f(E)dE$

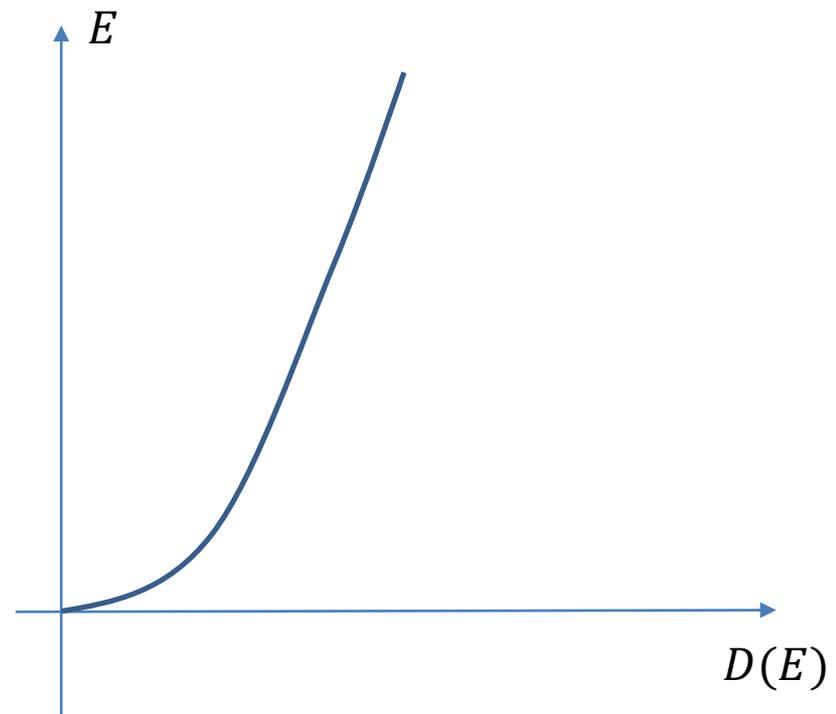
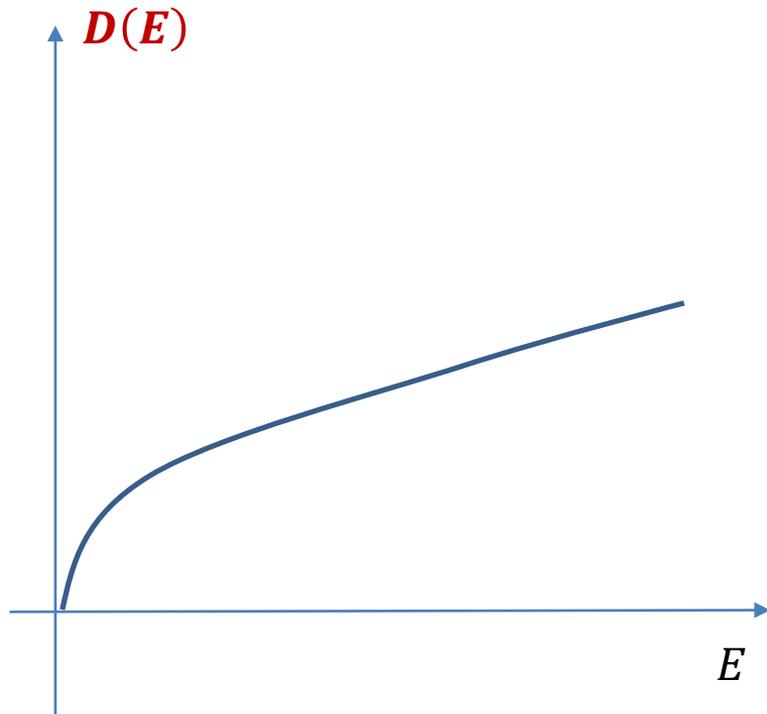
□ キャリア密度と有効状態密度の関係

□ 真性半導体キャリア密度

□ N型P型半導体キャリア密度

□ N型P型半導体キャリア密度

状態密度 $D(E)$



$$n(E) = \int_{E_1}^{E_n} D(E) \times \text{分布関数} dE$$

分布関数 $f(E)$

Maxwell-Boltzmann 分布関数

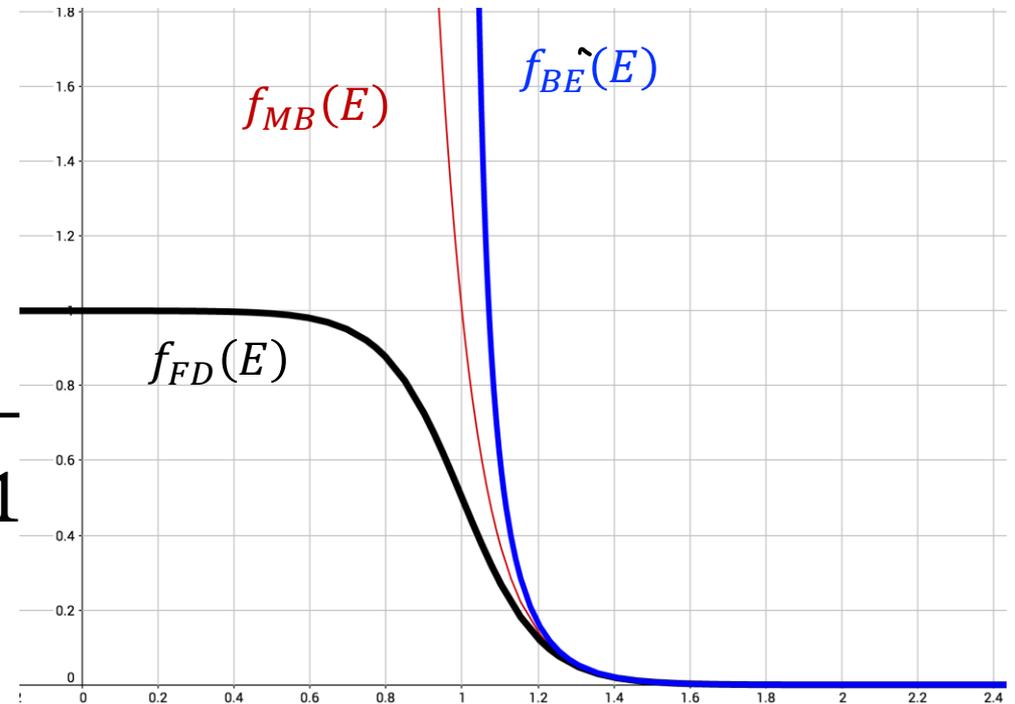
$$f_{MB}(E) = \frac{1}{\exp\left(\frac{E - u}{kT}\right)}$$

Fermi-Dirac の分布関数

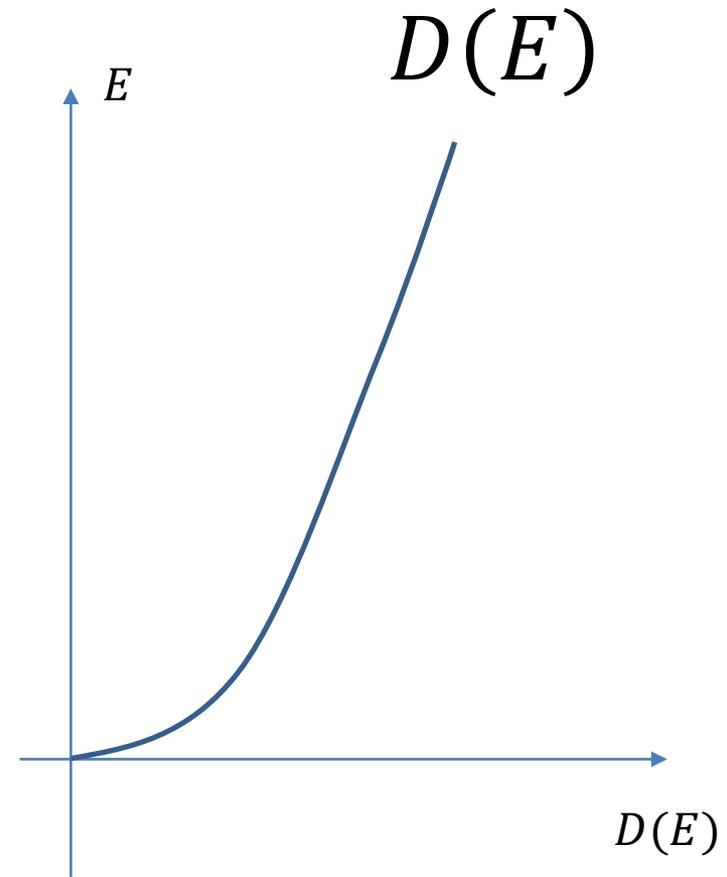
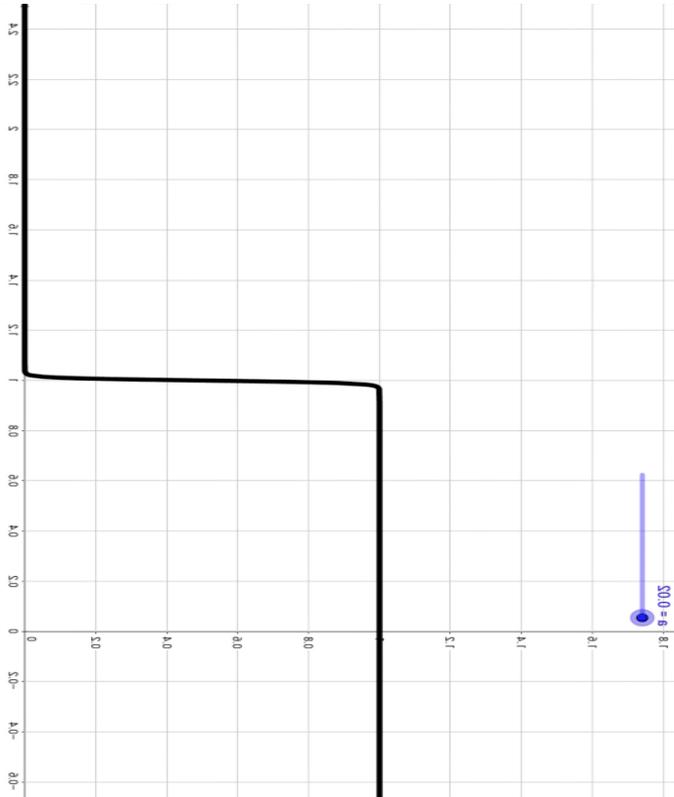
$$f_{FD}(E) = \frac{1}{\exp\left(\frac{E - u}{kT}\right) + 1}$$

Bose-Einstein 分布関数

$$f_{BE}(E) = \frac{1}{\exp\left(\frac{E - u}{kT}\right) - 1}$$



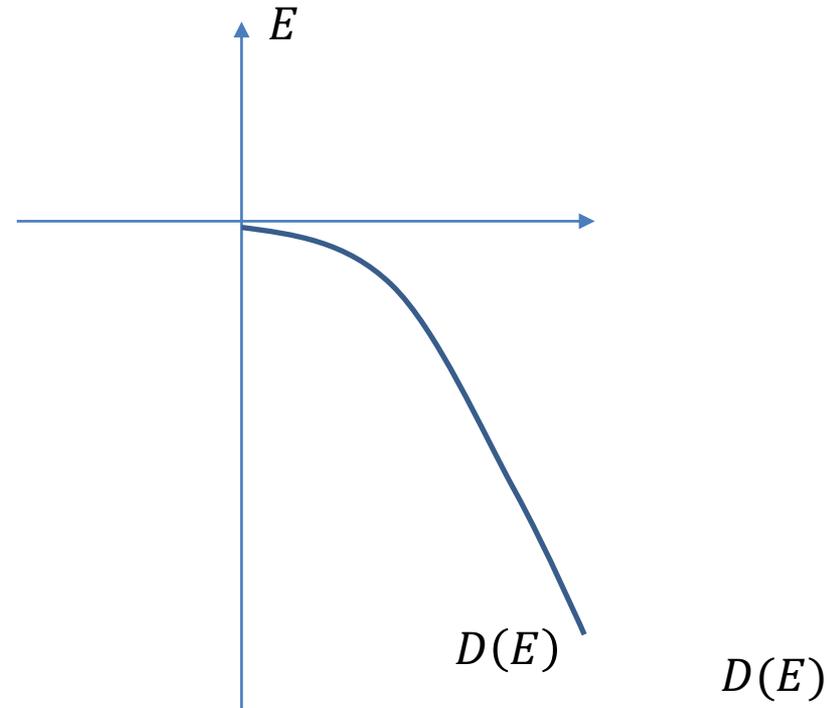
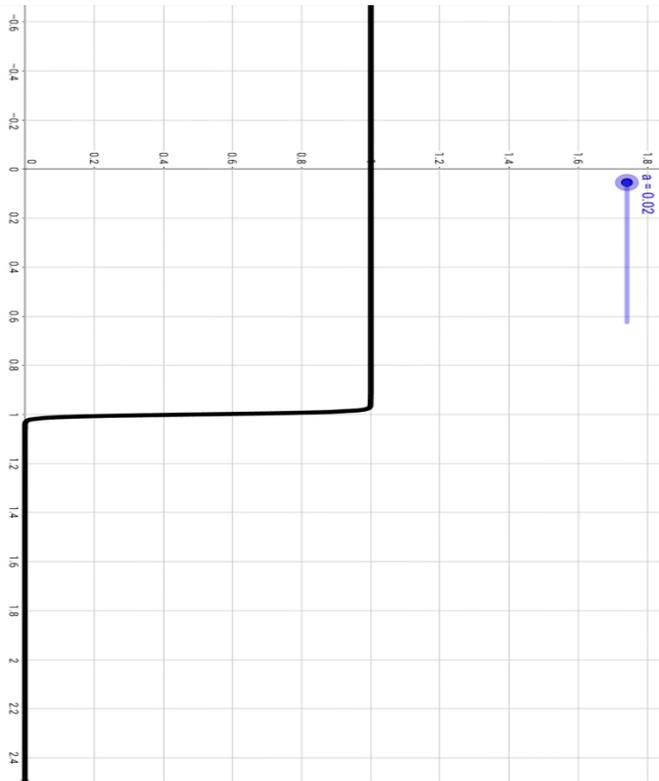
状態密度 × 分布関数：電子

 $f_n(E)$


$$n(E) = \int_{E_c}^{E_n} D(E) \times f(E) = dE$$

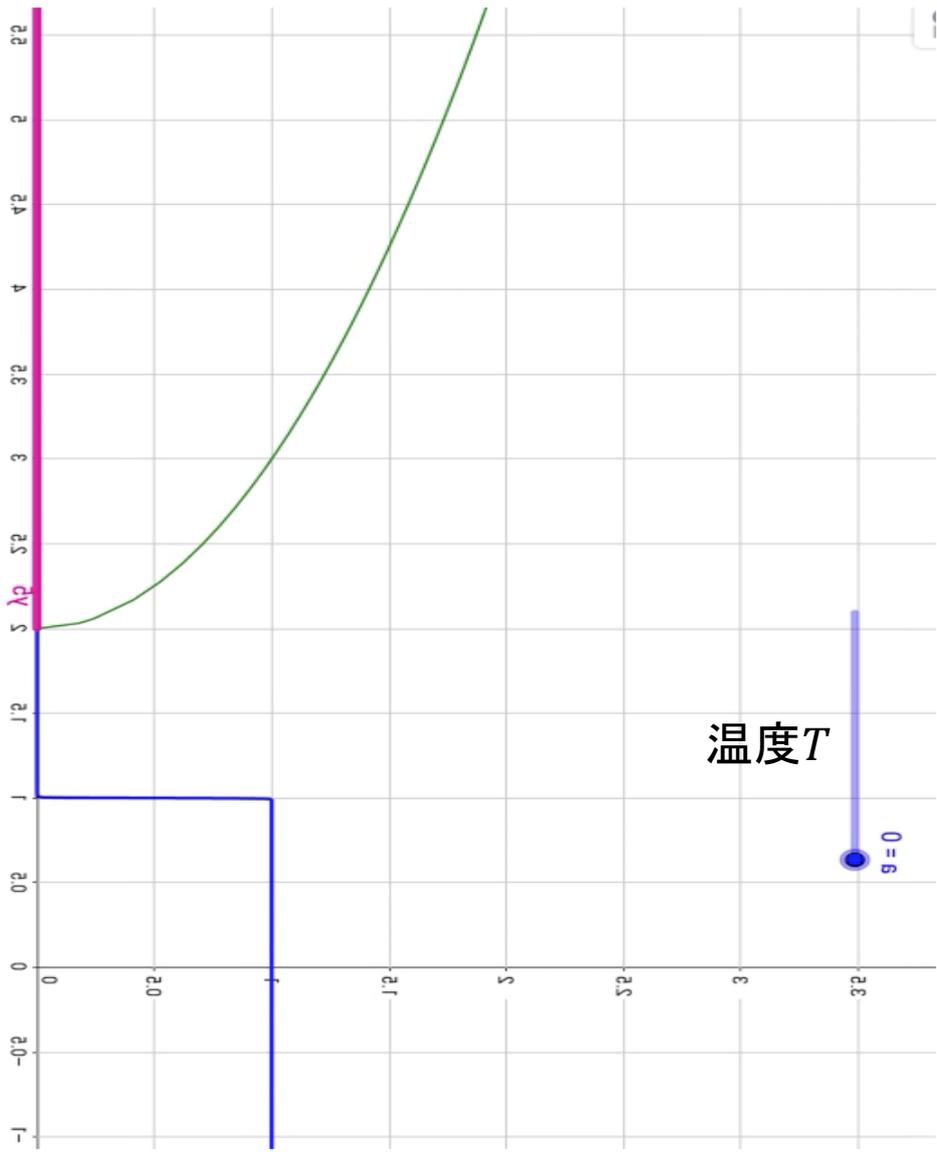
状態密度 × 分布関数: ホール

$$f_p(E) = 1 - f_n(E)$$

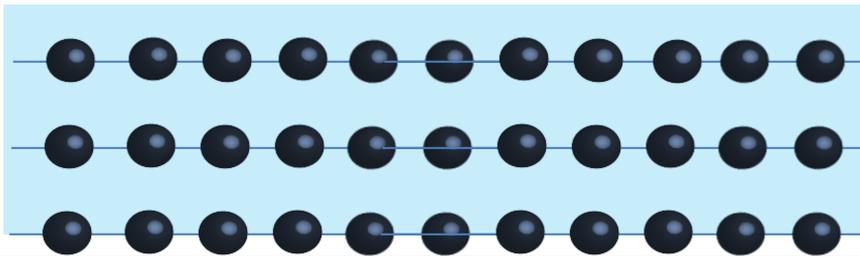


$$p(E) = \int_{E_1}^{E_n} D(E) \times f_p(E) = dE$$

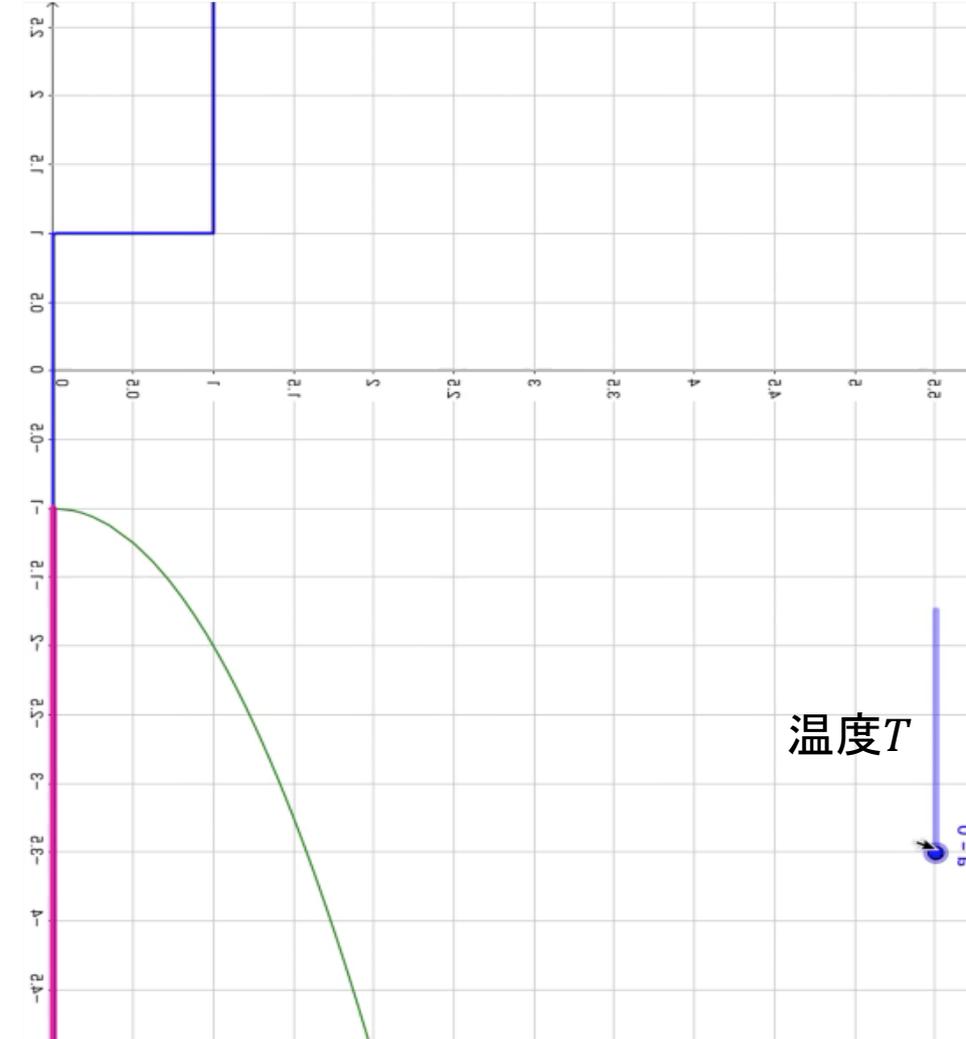
状態密度 × 分布関数 $D(E) \times f(E) ::$ 真性半導体 \rightarrow 電子



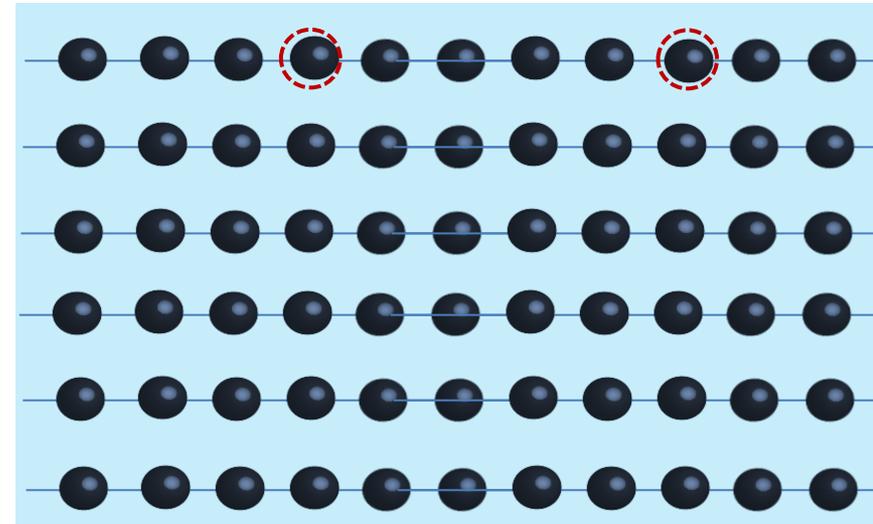
熱励起



状態密度 \times 分布関数 $D(E) \times f(E)$: 真性半導体 \rightarrow ホール



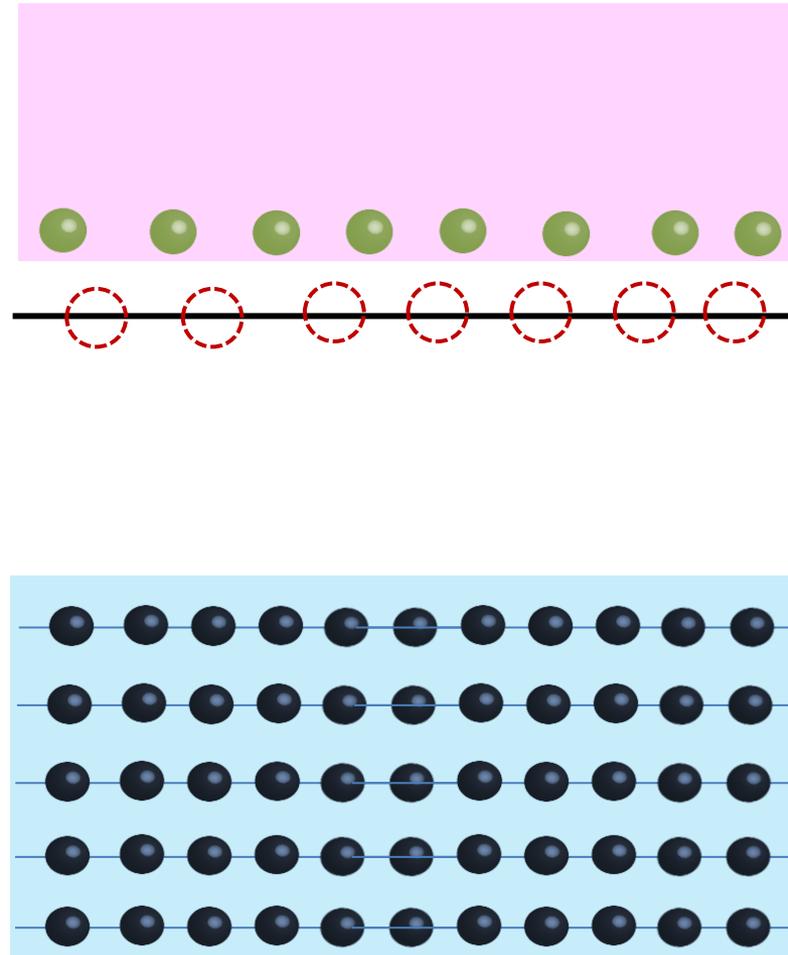
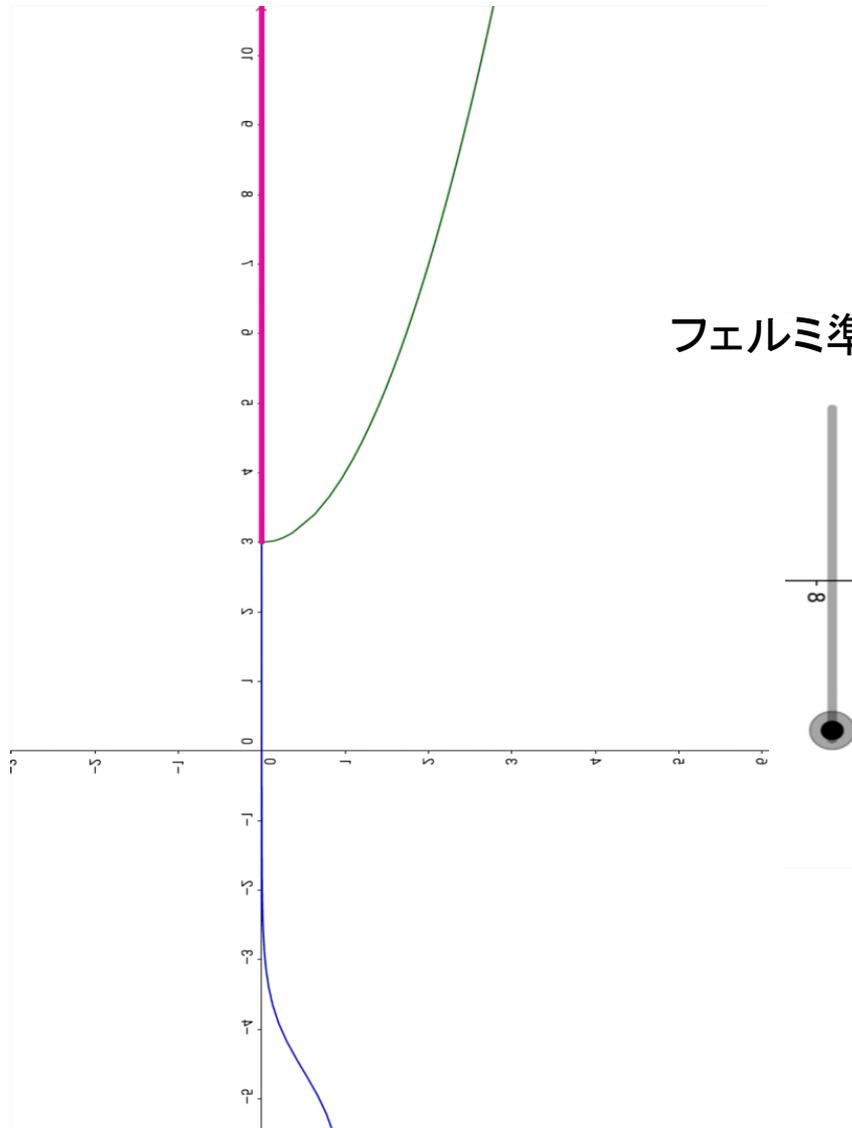
熱励起



状態密度 × 分布関数 $D(E) \times f(E)$: N型半導体

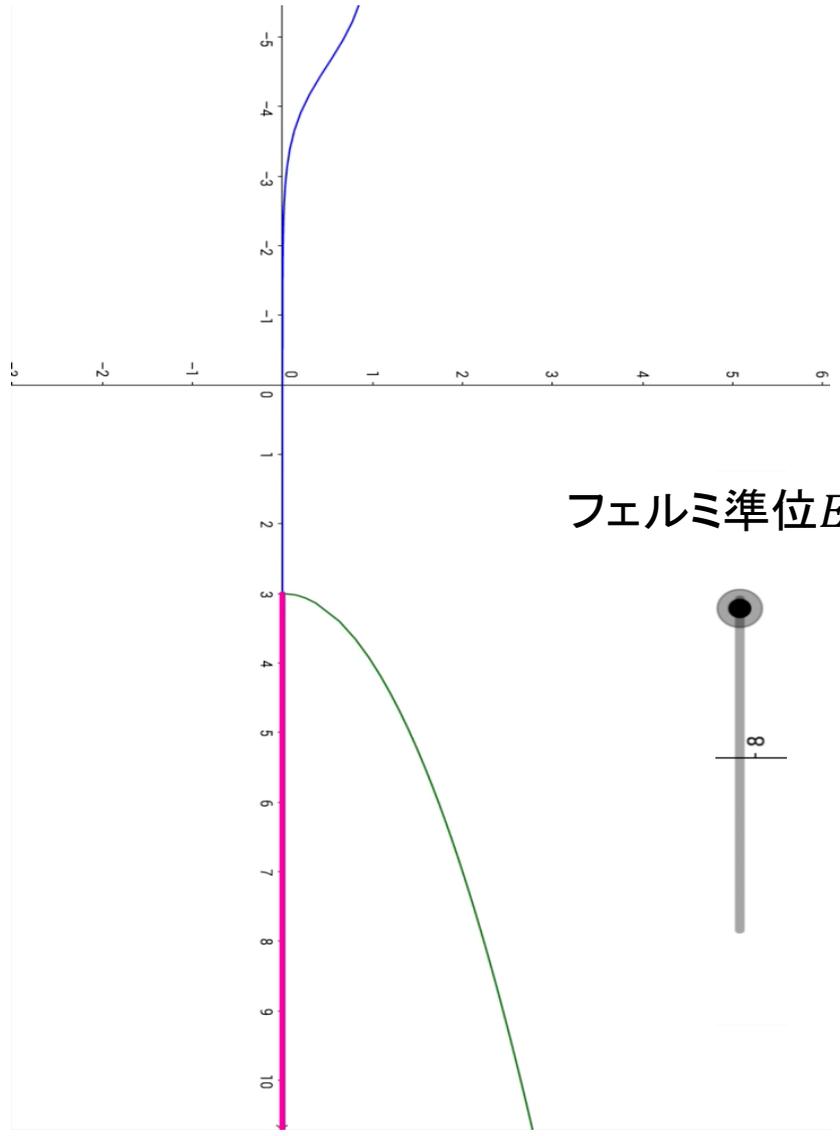
$$kT = 0.025(300K)$$

温度を300Kに固定する

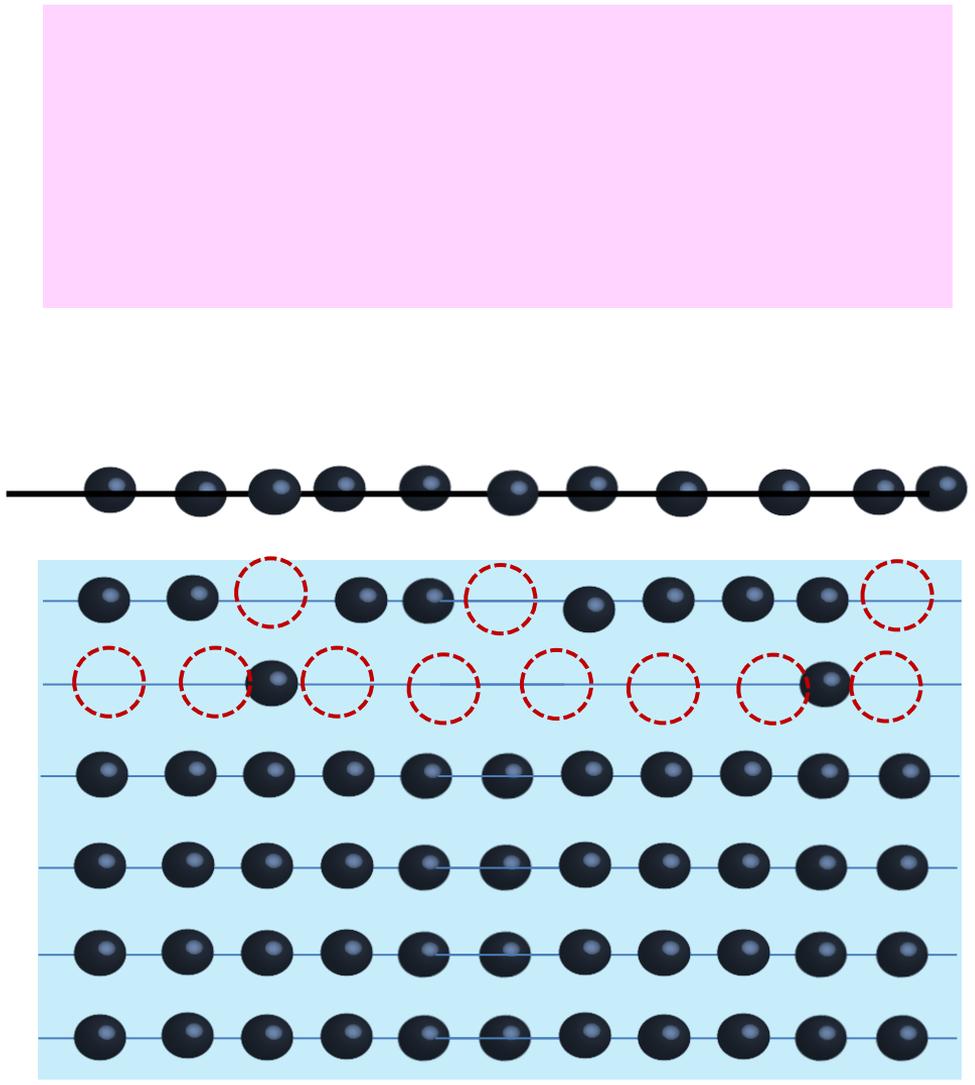
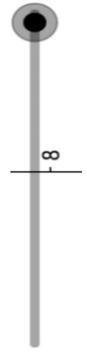


状態密度 × 分布関数 $D(E) \times f(E)$: P型半導体

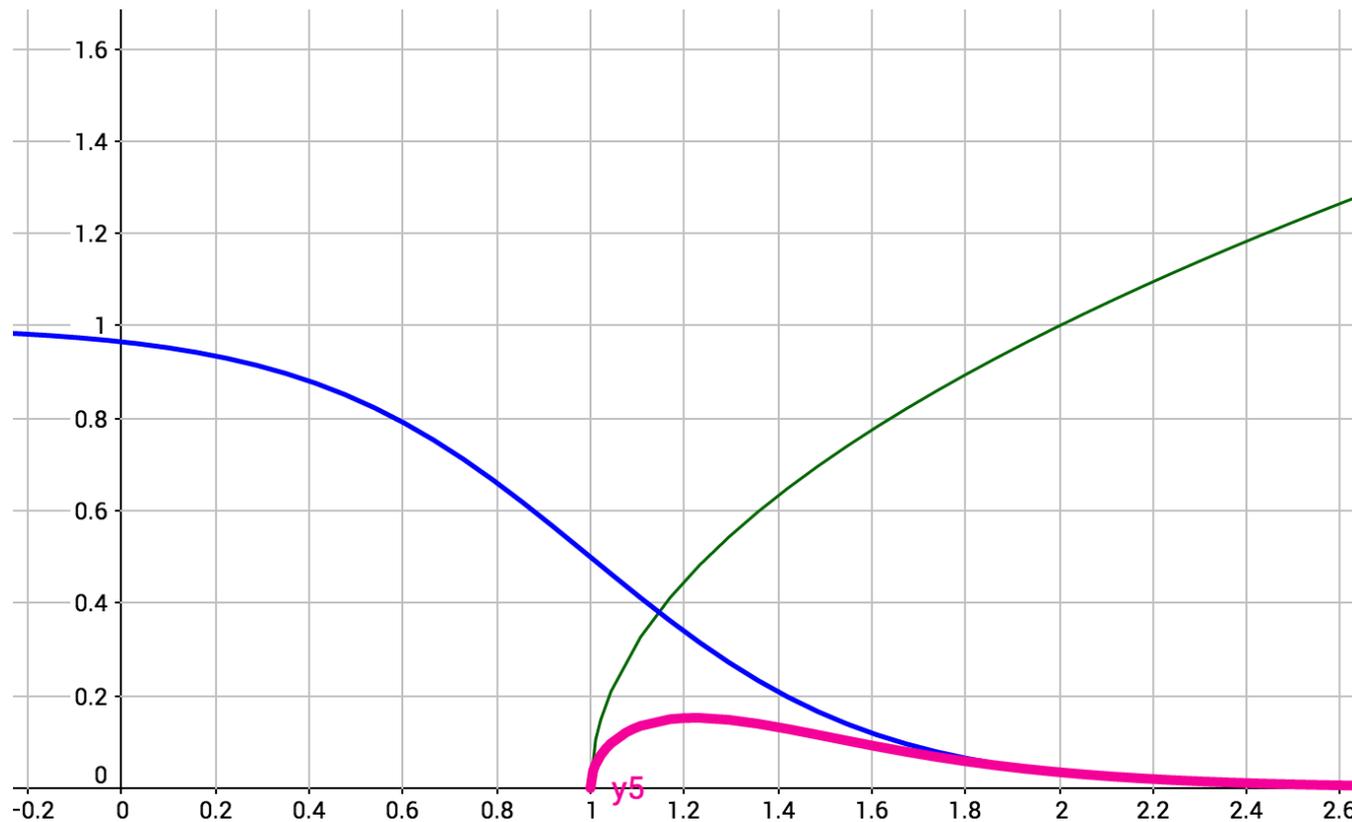
温度を300Kに固定する



フェルミ準位 E_f



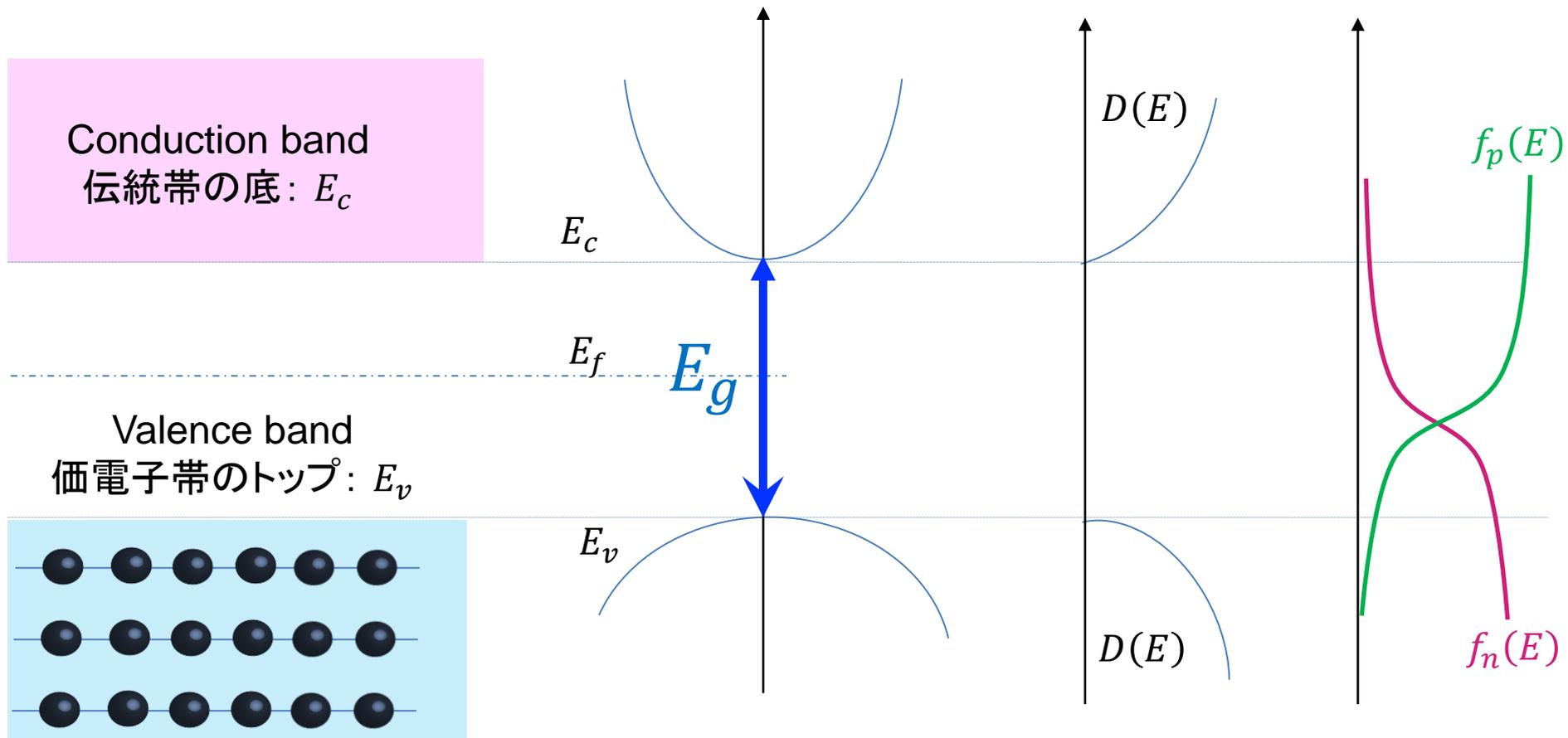
キャリア密度の積分計算:



$$n(E) = \int_{E_1}^{E_n} D(E) \times f(E) dE$$

キャリア密度の積分計算：符号の定義

$$n(E) = \int_{E_c}^{\infty} \frac{1}{2\pi^2} \left(\frac{8\pi^2 m^*}{h^2} \right)^{\frac{3}{2}} (E - E_c)^{\frac{1}{2}} \times \frac{1}{\exp\left(\frac{E - E_f}{kT}\right) + 1} dE$$



電子密度の計算:

$$n = \int_{E_c}^{\infty} \frac{1}{2\pi^2} \left(\frac{8\pi^2 m_e^*}{h^2} \right)^{\frac{3}{2}} (E - E_c)^{\frac{1}{2}} \times \frac{1}{\exp\left(\frac{E - E_f}{kT}\right) + 1} dE$$

Fermi-Diracの分布 \rightarrow Boltzmann 分布

$$\frac{1}{\exp\left(\frac{E - E_f}{kT}\right) + 1} = \exp\left(-\frac{E - E_f}{kT}\right)$$

$$\exp\left(-\frac{E - E_f}{kT}\right) = \exp\left(-\frac{E - E_c + E_c - E_f}{kT}\right) = \exp\left(-\frac{E - E_c}{kT}\right) \exp\left(-\frac{E_c - E_f}{kT}\right)$$

$$\exp\left(-\frac{E - E_f}{kT}\right) = \exp\left(-\frac{E - E_c}{kT}\right) \exp\left(\frac{E_f - E_c}{kT}\right)$$

電子密度の計算:

$$n = \int_{E_c}^{\infty} \frac{1}{2\pi^2} \left(\frac{8\pi^2 m_e^*}{h^2} \right)^{\frac{3}{2}} (E - E_c)^{\frac{1}{2}} \exp\left(-\frac{E - E_c}{kT}\right) \exp\left(\frac{E_f - E_c}{kT}\right) dE$$

$$n = \int_{E_c}^{\infty} \frac{1}{2\pi^2} \left(\frac{8\pi^2 m_e^*}{h^2} \right)^{\frac{3}{2}} (E - E_c)^{\frac{1}{2}} \exp\left(-\frac{E - E_c}{kT}\right) dE * \exp\left(\frac{E_f - E_c}{kT}\right)$$

$$n = 2 \left(\frac{2\pi m_e^* kT}{h^2} \right)^{\frac{3}{2}} * \exp\left(\frac{E_f - E_c}{kT}\right)$$

$$n = N_c * \exp\left(\frac{E_f - E_c}{kT}\right)$$

伝導帯の有効状態密度 N_c :

$$N_c = 2 \left(\frac{2\pi m_e^* kT}{h^2} \right)^{\frac{3}{2}}$$

シリコンの場合:

$$N_c = 6.2 \times 10^{15} \cdot T^{\frac{3}{2}} [cm^{-3}]$$

$$N_c = 3.2 \times 10^{19} [cm^{-3}]: \text{室温 } 300 [K]$$

$$n = 3.2 \times 10^{19} \exp\left(\frac{E_f - E_c}{kT}\right)$$

有効状態密度 N_c とキャリア濃度の関係:

$$n(E) = N_c \exp\left(\frac{E_f - E_c}{kT}\right)$$

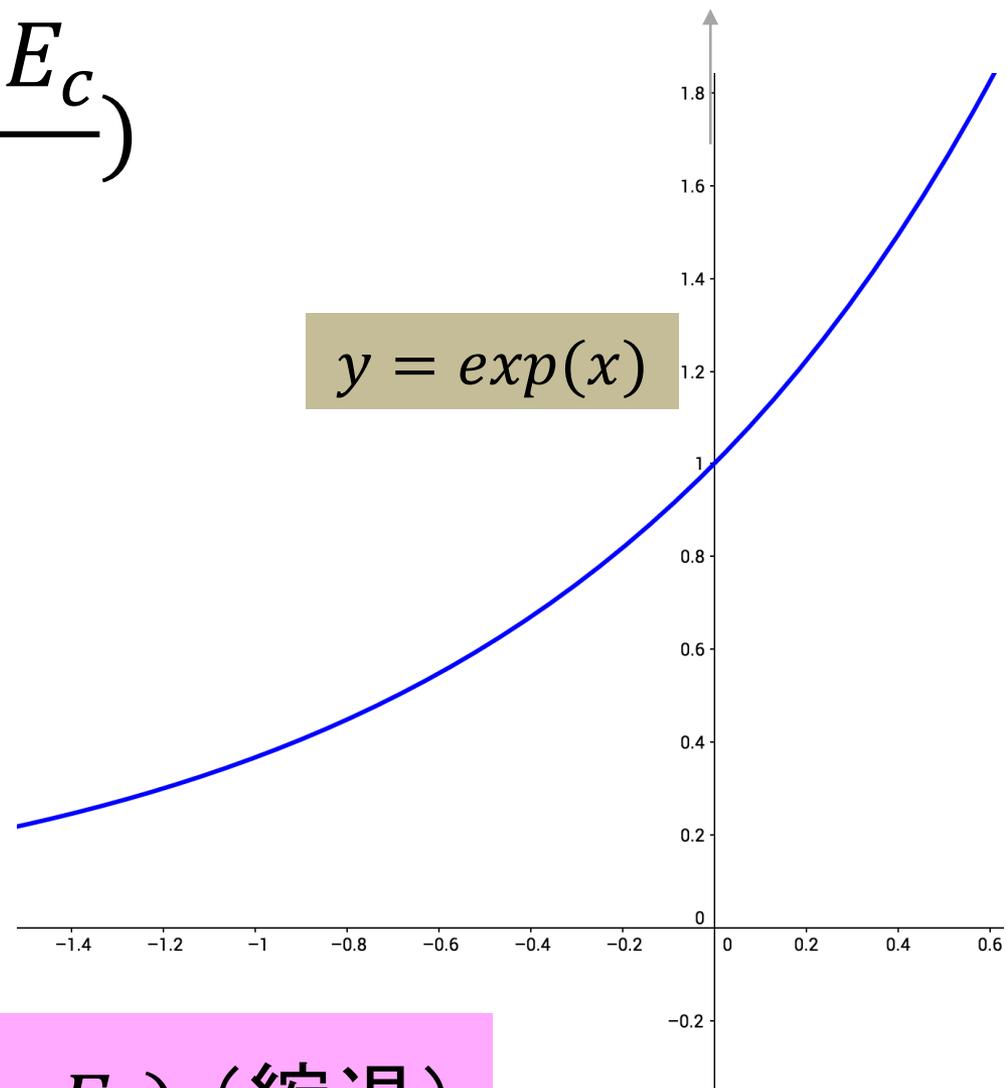
$$E_f - E_c \leq 0$$

$$\exp\left(\frac{E_f - E_c}{kT}\right) : 0 \sim 1$$

考慮しない

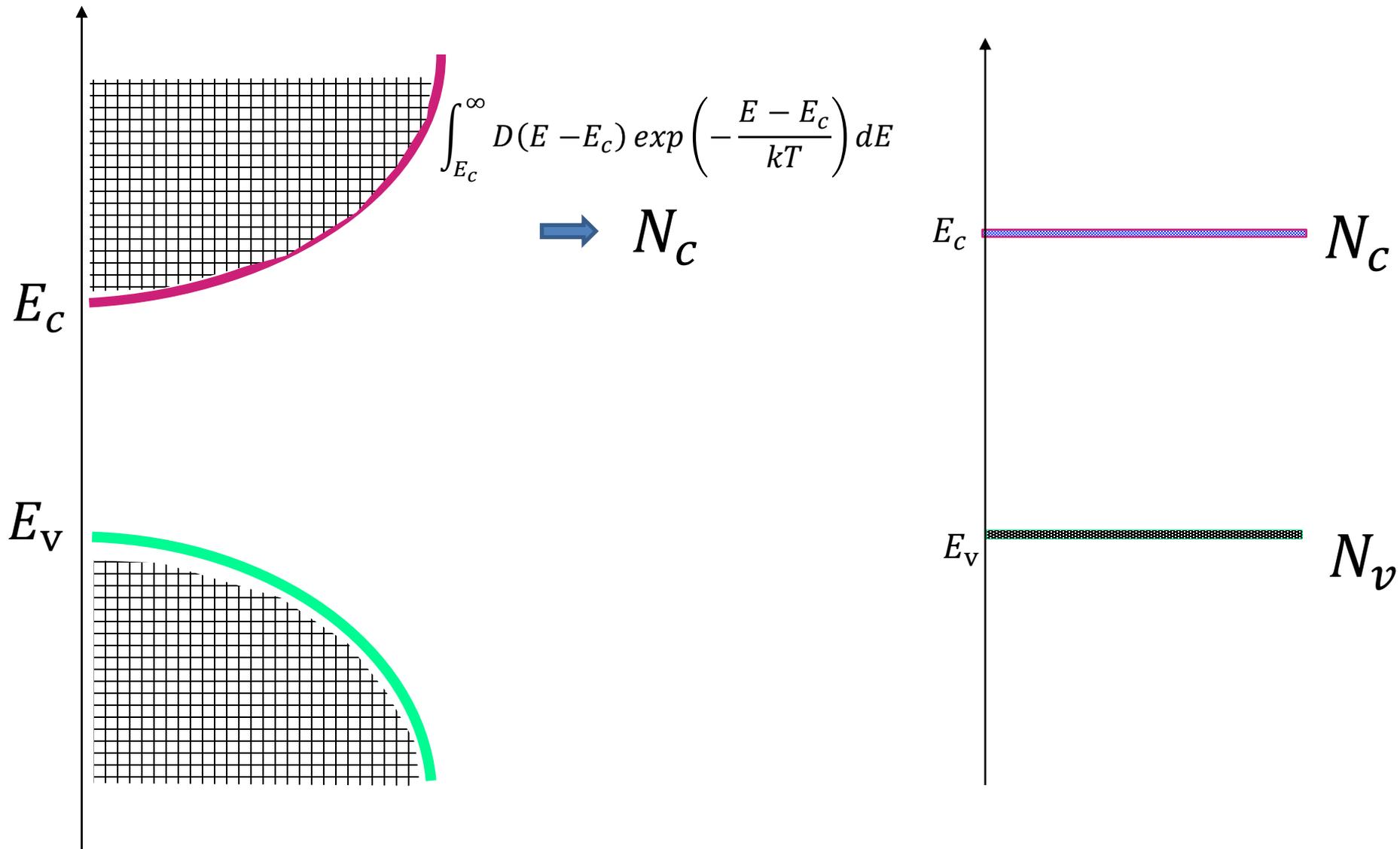
$$E_f - E_c > 0$$

$$\exp\left(\frac{E_f - E_c}{kT}\right) \propto \infty \quad (E_f > E_c) \quad (\text{縮退})$$

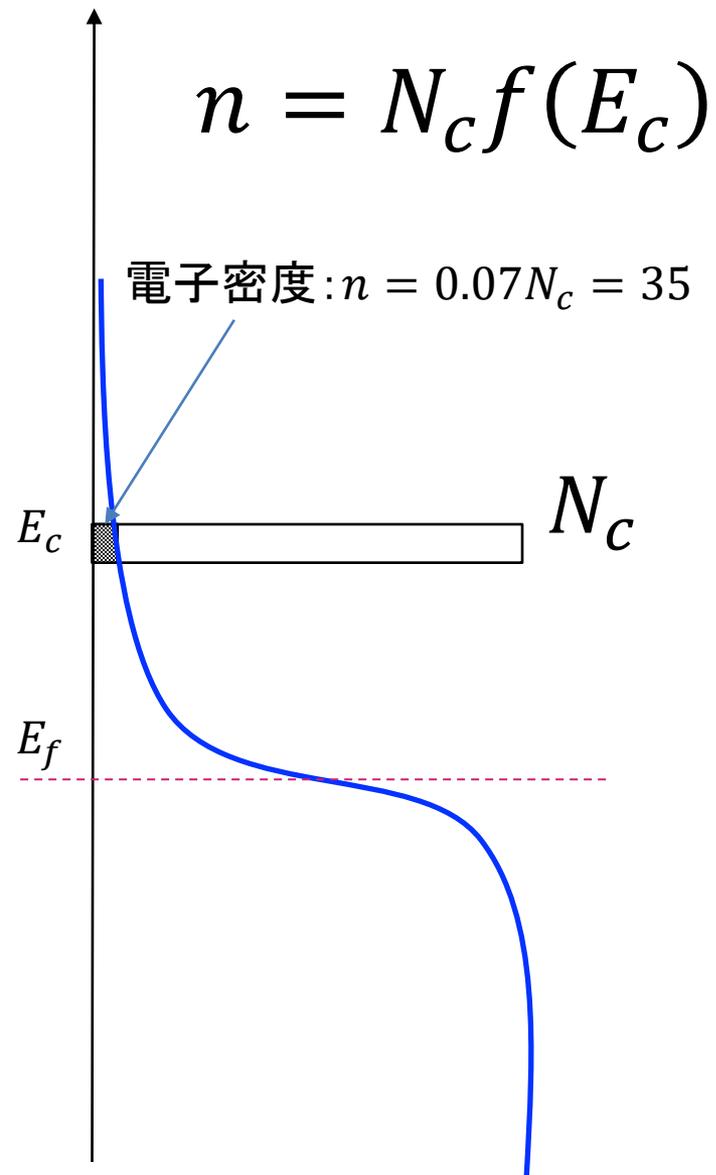
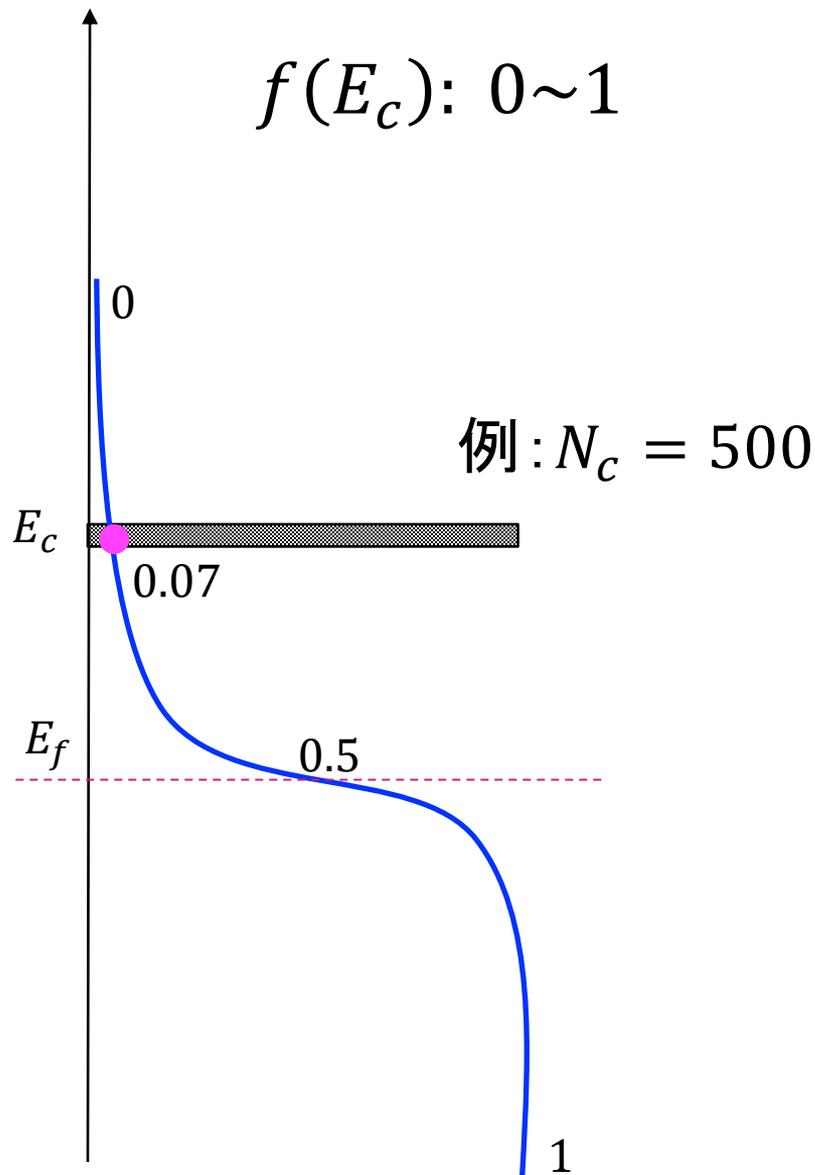


有効状態密度 N_C と状態密度の関係:

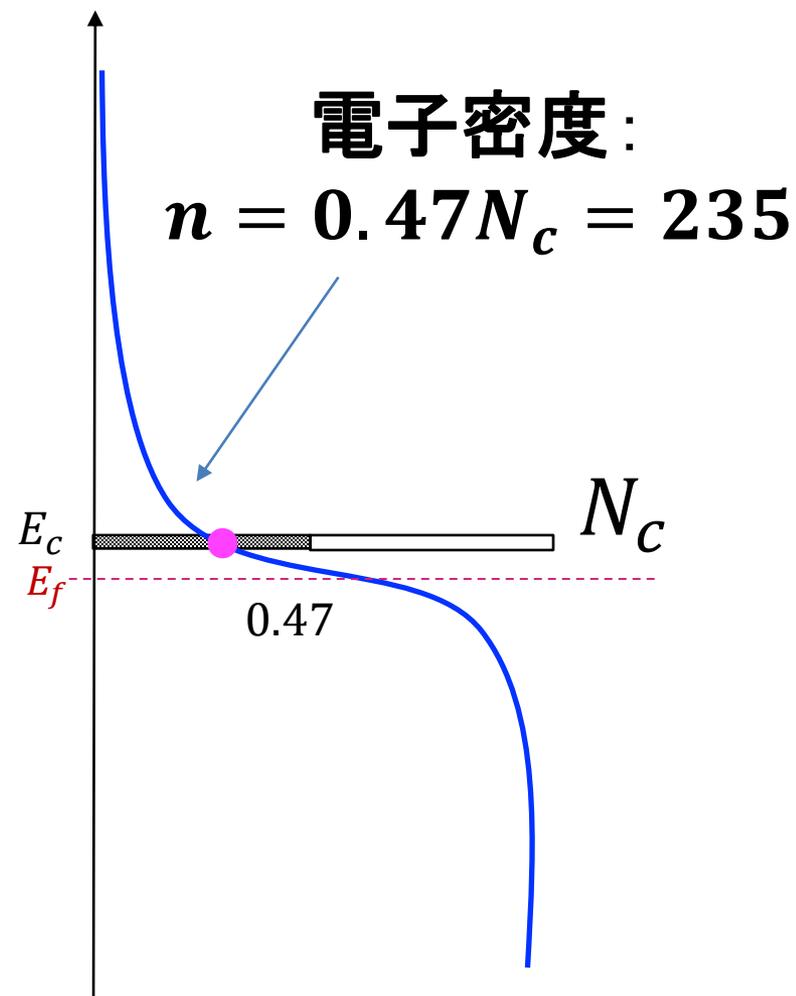
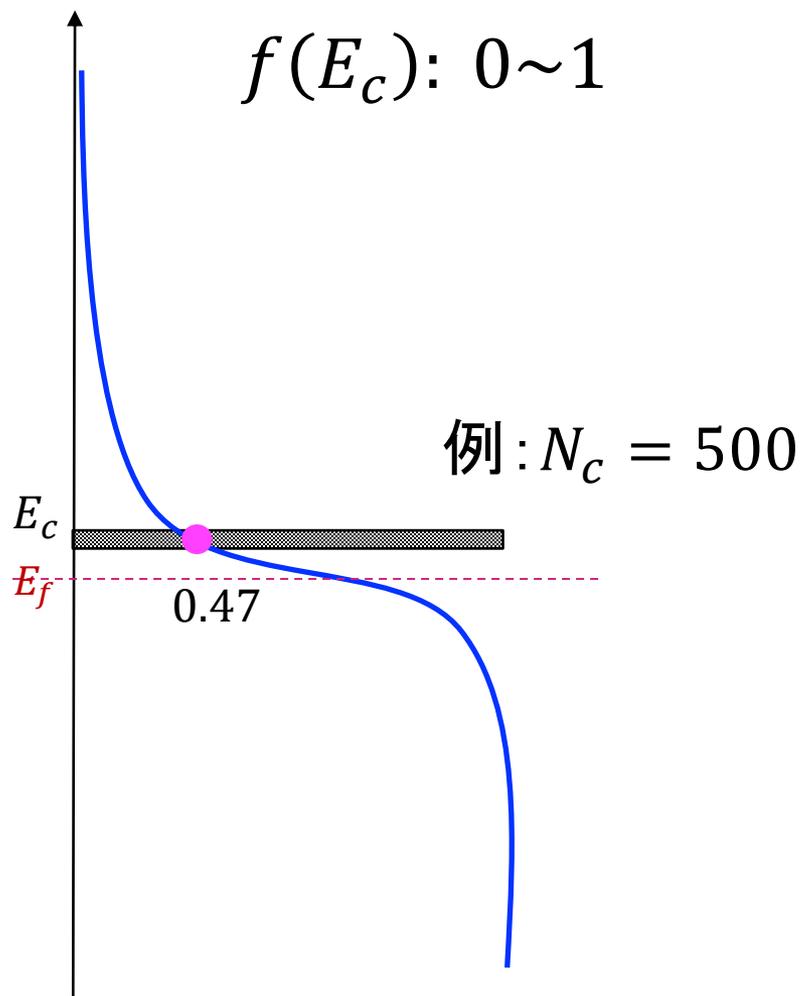
$D(E)$



電子密度計算プロセスの描像: $N_c f(E_c)$

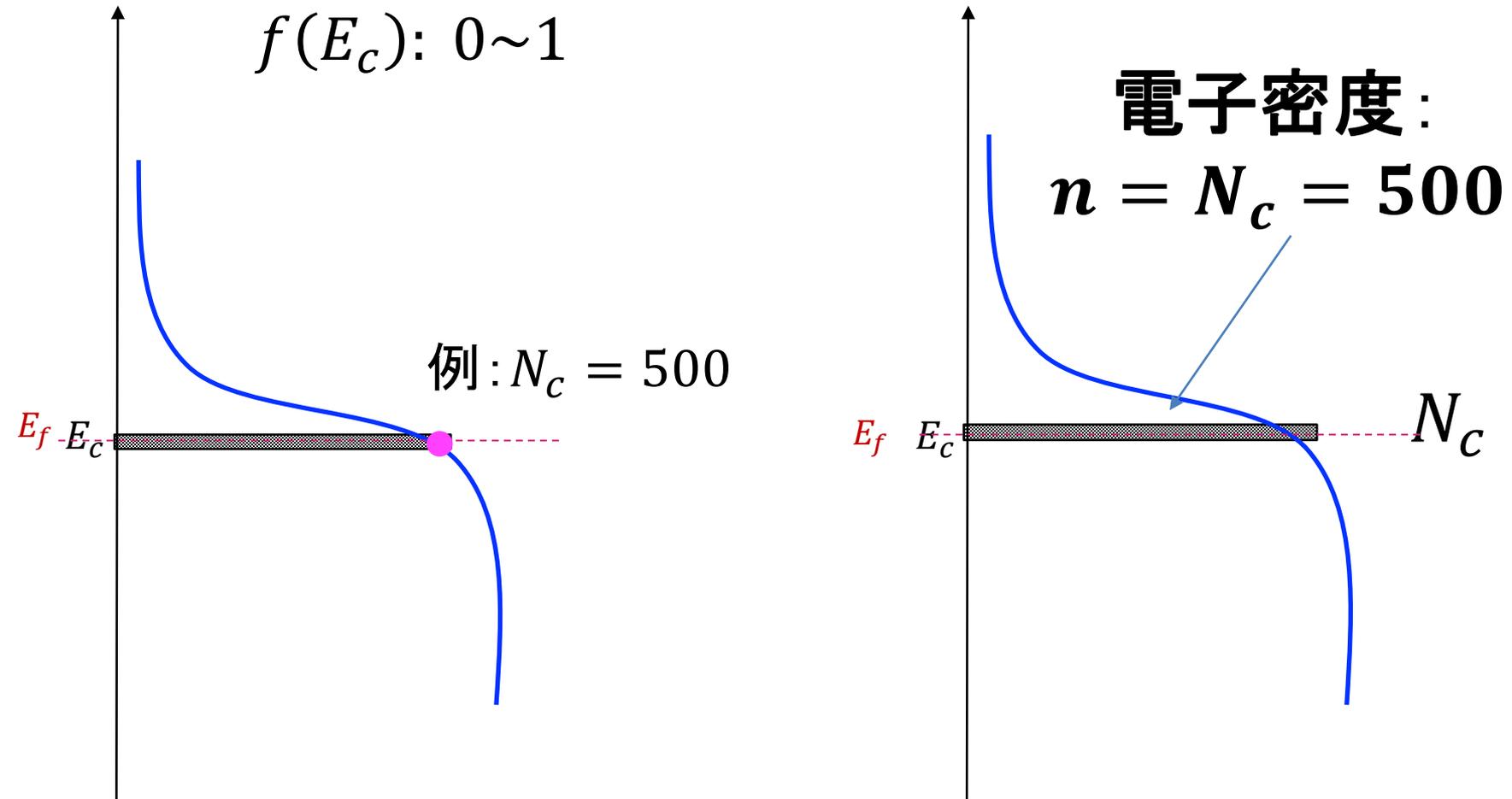


電子密度計算プロセスの描像: $N_c f(E_c)$



$f(E_c)$: E_c が固定なので、電子濃度を調整するのはフェルミ準位になる。
逆に、電子濃度がわかるとフェルミ準位がどこにあるかわかる。

電子密度計算プロセスの描像: $N_c f(E_c)$



$f(E_c)$: E_c が固定なので、フェルミ準位が E_c と等しくなると、
電子濃度が有効状態密度 N_c と等しくなる

電子密度計算例:

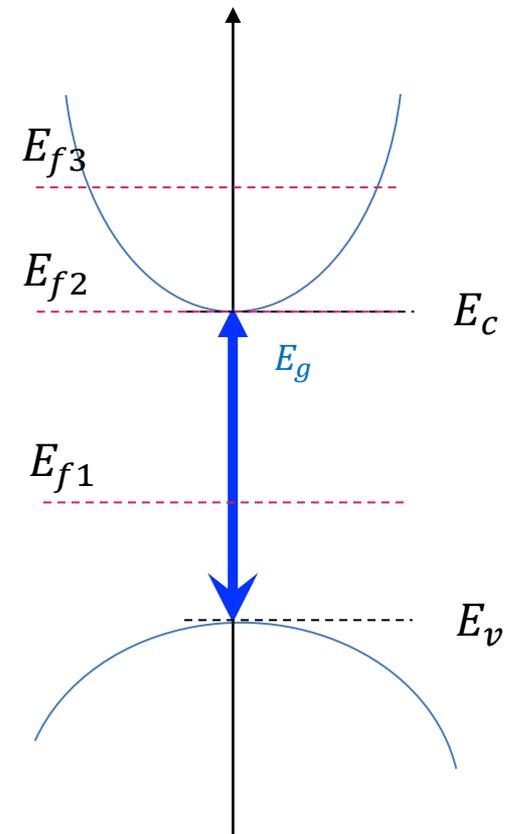
$$n = N_c \exp\left(\frac{E_f - E_c}{kT}\right)$$

$$n = 3.2 \times 10^{19} \exp[(E_f - E_c) * 40]$$

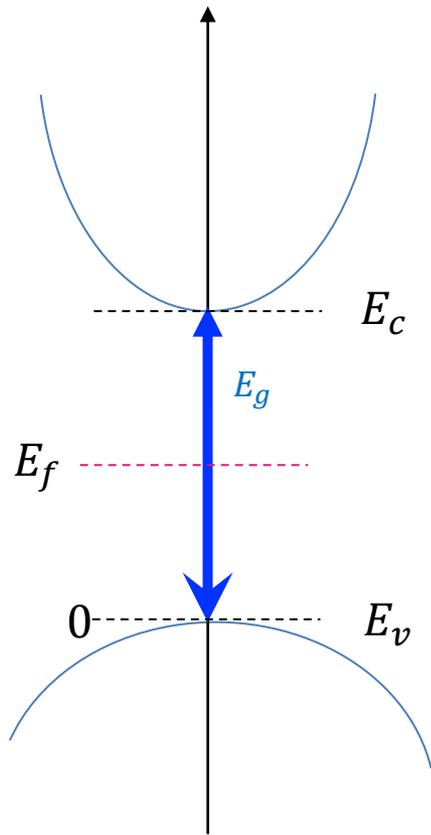
$$E_{f3} - E_c > 0 \quad n > 3.2 \times 10^{19} \text{ (縮退)}$$

$$E_{f2} - E_c = 0 \quad n = 3.2 \times 10^{19}$$

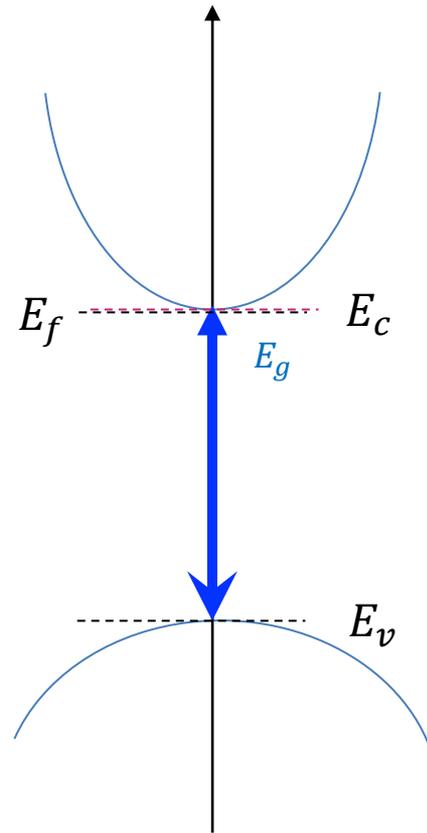
$$E_{f1} - E_c < 0 \quad n < 3.2 \times 10^{19}$$



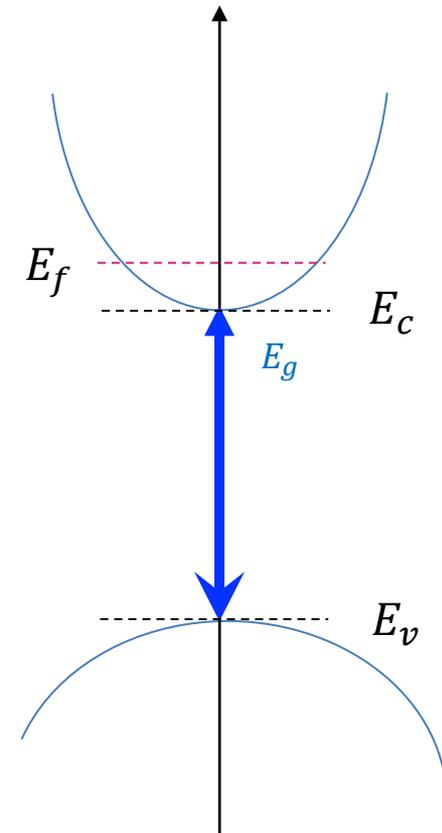
フェルミ準位とキャリア濃度の計算例:



$$n < N_c$$



$$n = N_c$$



$n > N_c$ (縮退)
考慮しない

ホール密度の計算:

$$p = \int_{-\infty}^{E_V} \frac{1}{2\pi^2} \left(\frac{8\pi^2 m_h^*}{h^2} \right)^{\frac{3}{2}} (E_V - E)^{\frac{1}{2}} \times \left\{ 1 - \frac{1}{\exp\left(\frac{E - E_f}{kT}\right) + 1} \right\} dE$$

ホールの分布関数 f_p : $1 - f_n$

Fermi-Diracの分布 \rightarrow Boltzmann 分布

$$p = N_V \exp\left(-\frac{E_V - E_f}{kT}\right)$$

$$N_V = 2 \left(\frac{2\pi m_h^* kT}{h^2} \right)^{\frac{3}{2}}$$

価電子帯の有効状態密度 N_V :

$$N_V = 2 \left(\frac{2\pi m_h^* kT}{h^2} \right)^{\frac{3}{2}}$$

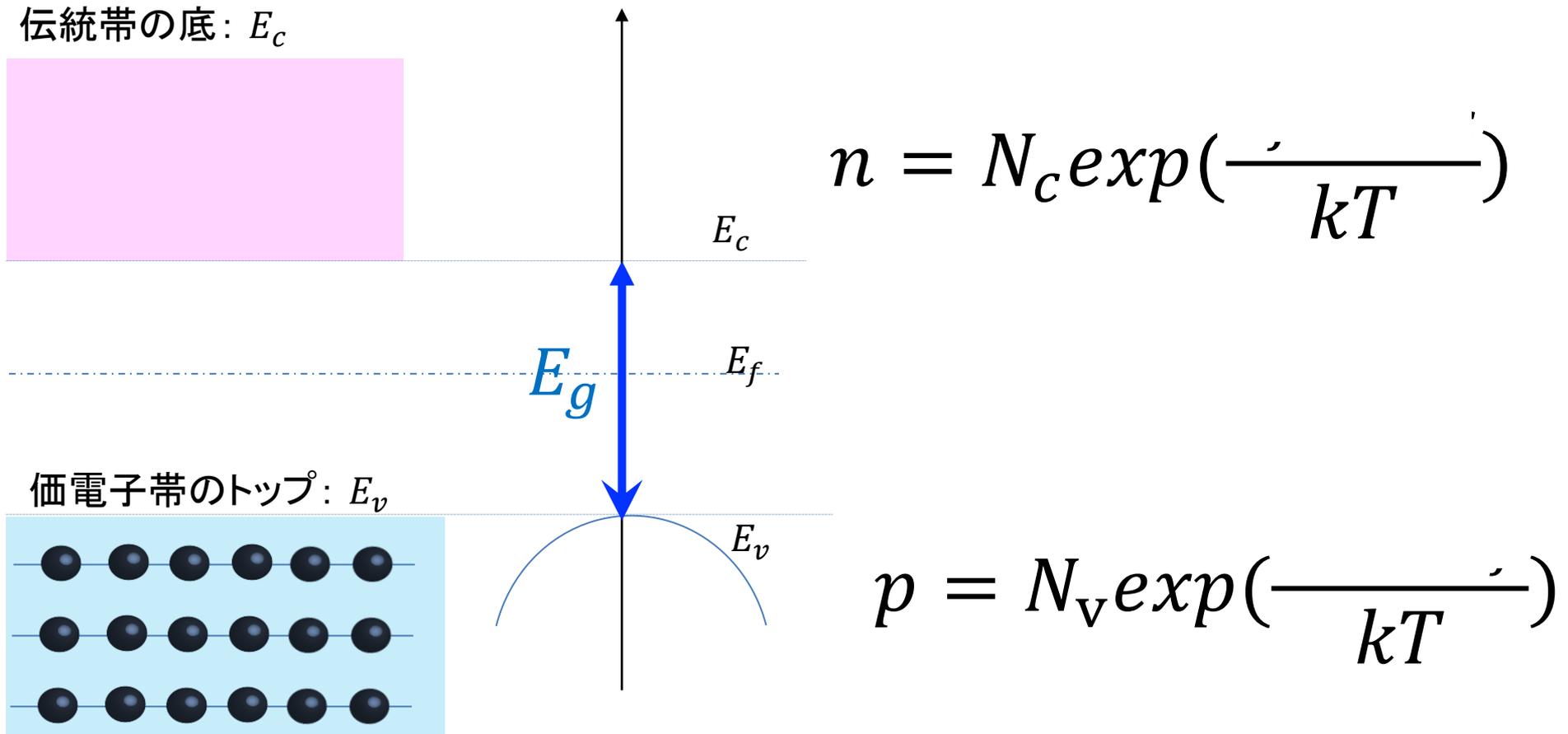
シリコンの場合:

$$N_V = 3.5 \times 10^{15} \cdot T^{\frac{3}{2}} [cm^{-3}]$$

$$N_V = 1.82 \times 10^{19} [cm^{-3}]: \text{室温} \\ 300[K]$$

$$p = 1.82 \times 10^{19} \exp\left(-\frac{E_g}{kT}\right)$$

不純物添加無し我真性半導体の電子とホール密度



真性半導体 $n = p$

熱平衡と非熱平衡とフェルミ準位:

熱平衡: 光照射なし、バイアス電圧なし、温度効果だけを考慮する。

真性半導体・P型半導体とN型半導体において:

$$E_{fn} = E_{fp} = E_f$$

熱平衡ではない時:

光照射あり: 太陽電池、

バイアス電圧あり: 太陽電池、LED。

真性半導体においては:

$$E_{fn} = E_{fp} = E_f$$

P型半導体とN型半導体においては:

$$E_{fn} \neq E_{fp} \neq E_f$$

真性半導体のフェルミ準位:

真性半導体 $n = p$

$$\frac{n}{p} = 1$$

$$\frac{N_c \exp\left(\frac{E_f - E_c}{kT}\right)}{N_v \exp\left(\frac{E_v - E_f}{kT}\right)} = 1 \quad \rightarrow \quad \frac{N_c \exp\left(\frac{2E_f - E_c - E_v}{kT}\right)}{N_v} = 1$$

$$\exp\left(\frac{2E_f - E_c - E_v}{kT}\right) = \frac{N_v}{N_c}$$

真性半導体のフェルミ準位:

$$\frac{2E_f - E_c - E_v}{kT} = \ln \frac{N_v}{N_c}$$

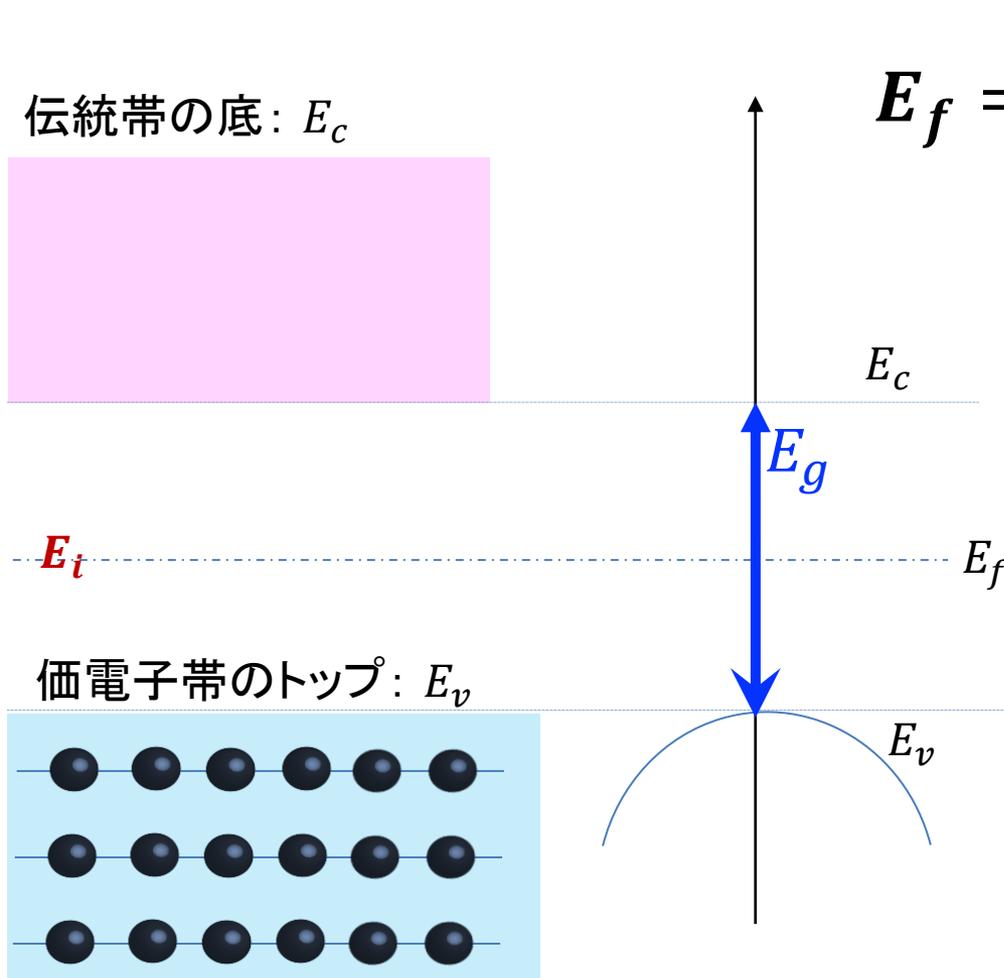


$$2E_f = E_c + E_v + kT \ln \frac{N_v}{N_c}$$



$$E_f = \frac{E_c + E_v}{2} + \frac{1}{2} kT \ln \frac{N_v}{N_c}$$

真性半導体のフェルミ準位と禁制帯の中心準位との関係:



$$E_f = \frac{E_c + E_v}{2} + \frac{1}{2} kT \ln \frac{N_v}{N_c}$$

E_i : 禁制帯の中心準位

$$E_f = E_i + \frac{1}{2} kT \ln \frac{N_v}{N_c}$$

$$\frac{N_v}{N_c} \approx 1$$

$$E_f \approx E_i = \frac{E_c + E_v}{2}$$

熱平衡時真性半導体の pn 積

$$n = N_c \exp\left(\frac{E_f - E_c}{kT}\right)$$



$$p = N_v \exp\left(\frac{E_v - E_f}{kT}\right)$$



$$pn = N_c N_v \exp\left(\frac{E_v - E_c}{kT}\right)$$

真性半導体の真性キャリア密度 n_i :

$$pn = N_c N_v \exp\left(\frac{E_v - E_c}{kT}\right)$$

$$pn = N_c N_v \exp\left(\frac{-E_g}{kT}\right) \leftarrow \text{定数 } n_i^2$$

シリコンの真性キャリア密度 n_i :

$$n_i^2 = N_c N_v \exp\left(\frac{E_v - E_c}{kT}\right) = N_c N_v \exp\left(\frac{-E_g}{kT}\right)$$

$$n_i = \sqrt{N_c N_v} \exp\left(\frac{E_v - E_c}{2kT}\right) = \sqrt{N_c N_v} \exp\left(\frac{-E_g}{2kT}\right)$$

真性半導体の真性キャリア密度:

$$n_i^2 = N_c N_v \exp\left(\frac{-E_g}{kT}\right)$$

$$n_i = \sqrt{N_c N_v} \exp\left(\frac{-E_g}{2kT}\right)$$

$$N_c = 3.2 \times 10^{19}, \quad N_v = 1.8 \times 10^{19}, \quad E_g = 1.1[\text{eV}], \\ kT = 0.025[\text{eV}],$$

$$n_i^2 = 1.82 \times 3.2 \times 10^{38} \times 7.78 \times 10^{-20} \approx 1 \times 10^{20}$$

$$n_i \approx 1 \times 10^{10} [\text{cm}^{-3}]$$

真性半導体の真性キャリア密度:

$$pn = n_i^2$$

真性半導体
N型半導体
P型半導体 } 成立

真性半導体

$$n = p$$

$$n = p = n_i$$

N型半導体 P型半導体

$$n \neq p$$

$$n \neq p \neq n_i$$

熱平衡不純物半導体のキャリア密度:

熱平衡: 光照射なし、バイアス電圧なし、
温度効果だけを考慮する。

真性半導体・P型半導体とN型半導体において:

$$E_{fn} = E_{fp} = E_f$$

初級編

完全イオン化近似@室温

$p \ll n$ あるいは $n \ll p$ 近似

中級編

部分イオン化近似→温度依存性あり

$p \ll n$ あるいは $n \ll p$ 近似

上級編

部分イオン化近似→温度依存性あり

p と n を近似無しで $pn = n_i^2$ を使って計算

不純物半導体のキャリア密度：N型半導体の電子密度

最も簡単な場合：完全イオン化近似@室温(初級編)

$$n^- = p^+ + N_d^+$$

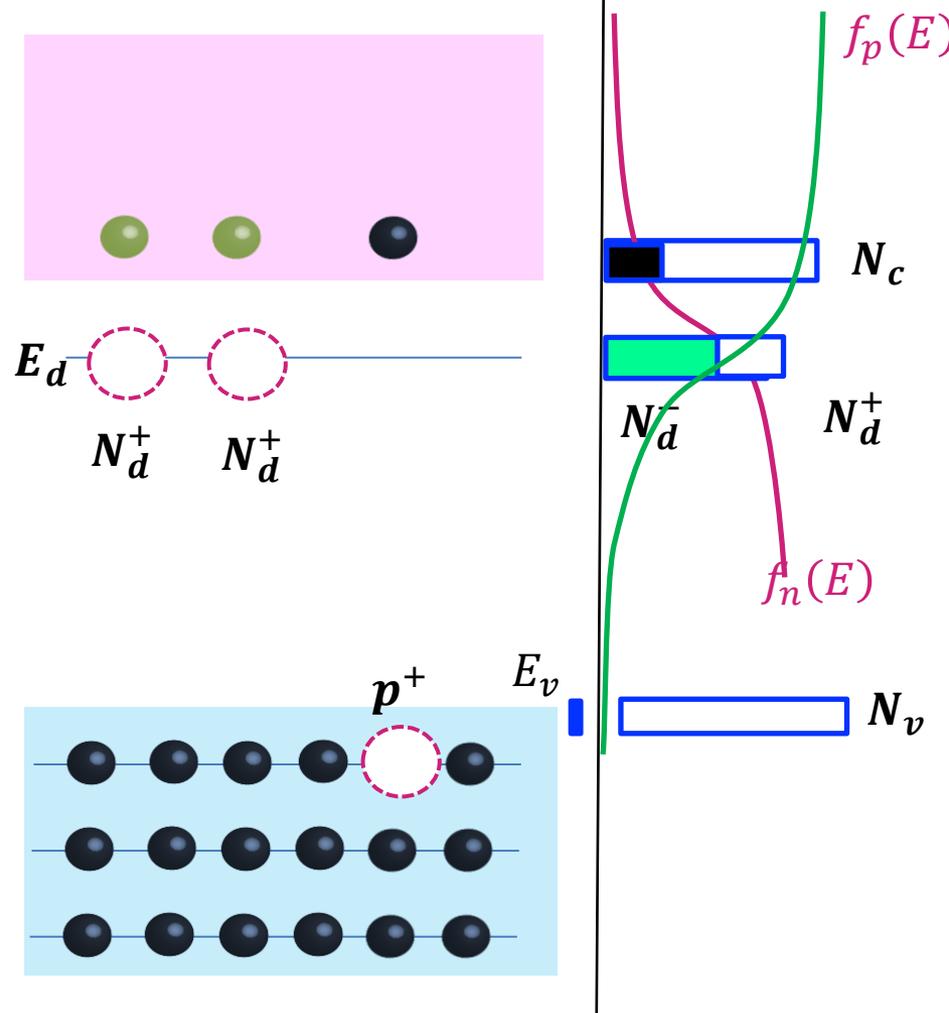
$$n = p + N_d^+$$

$$p \ll n$$

$$n \approx N_d^+ = N_d$$

$$pn = n_i^2$$

$$p = \frac{n_i^2}{N_d}$$



不純物半導体のキャリア密度: P型半導体のホール密度

最も簡単な場合: 完全イオン化近似@室温(初級編)

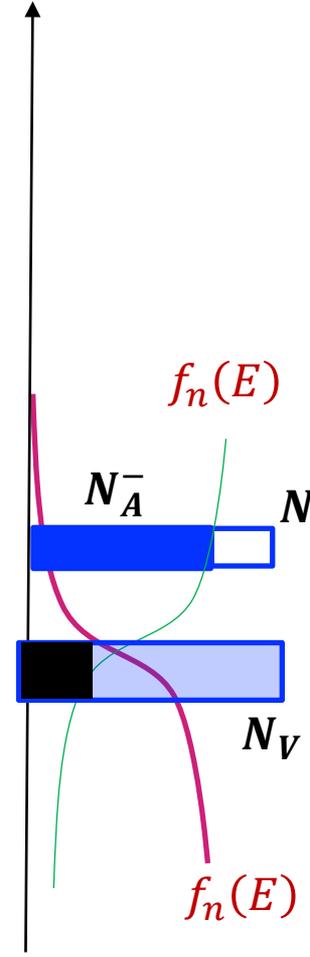
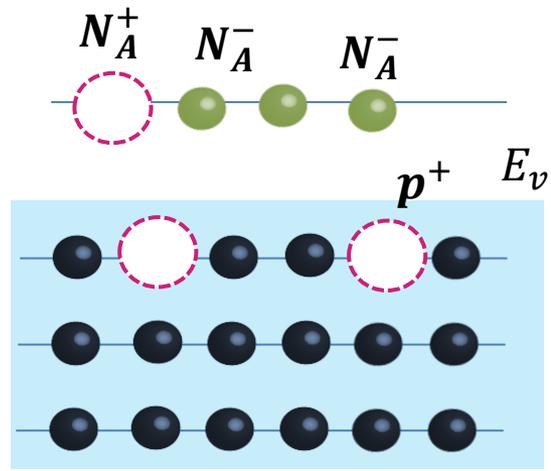
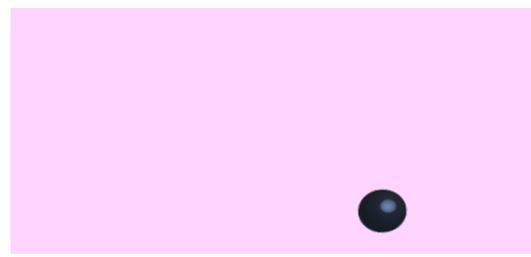
$$p^+ = n^- + N_A^-$$

$$p = n + N_A^-$$

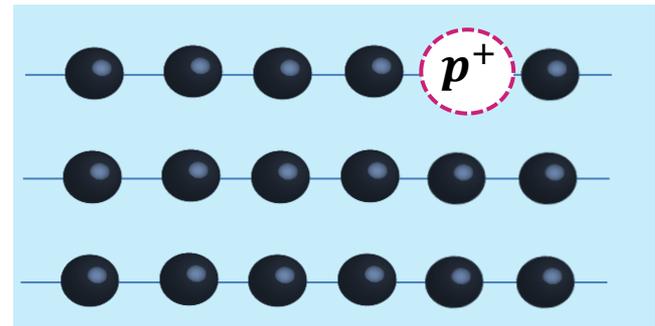
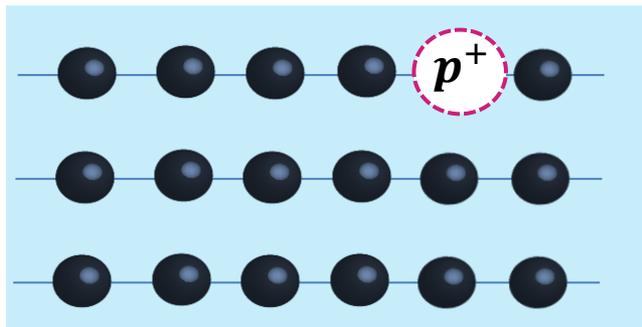
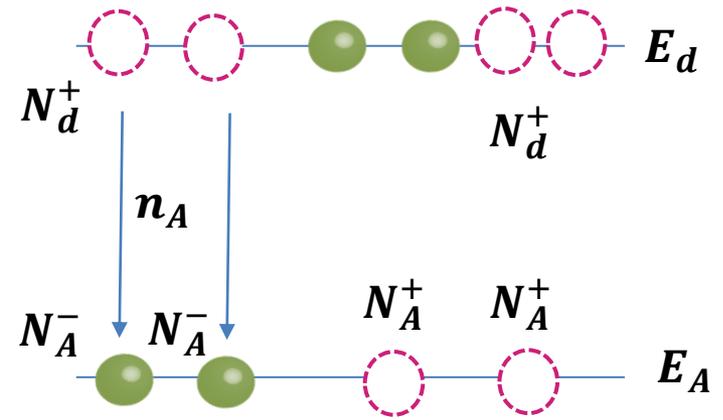
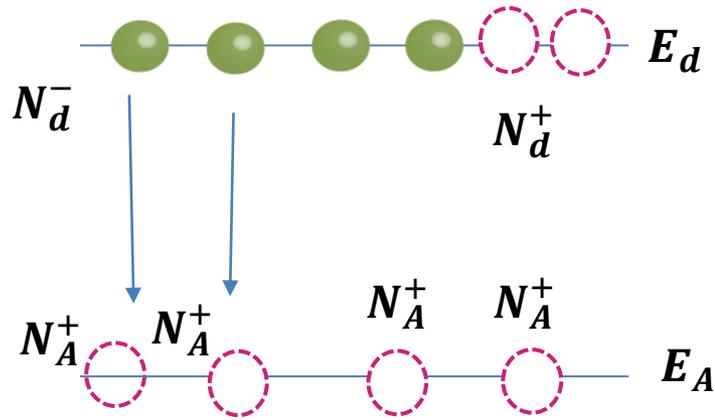
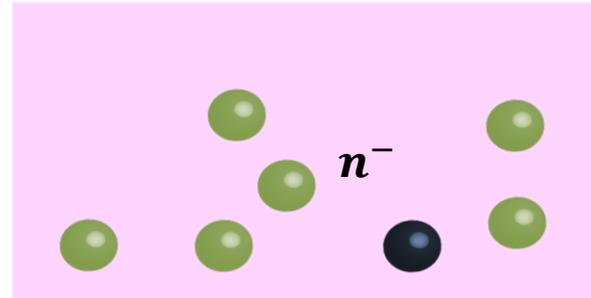
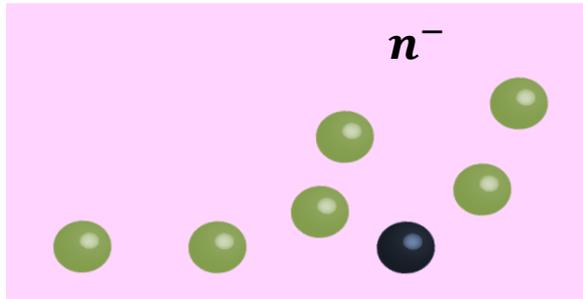
$$n \ll p$$

$$p \approx N_A^- = N_A$$

$$n = \frac{n_i^2}{p}$$



N型半導体+少量のP型不純物



N型半導体+少量のP型不純物

最も簡単な場合：完全イオン化近似@室温

$$p^+ + N_D^+ = n^- + N_A^-$$

$$N_A^- = n_A = N_A$$

$$p + N_D = n + N_A$$

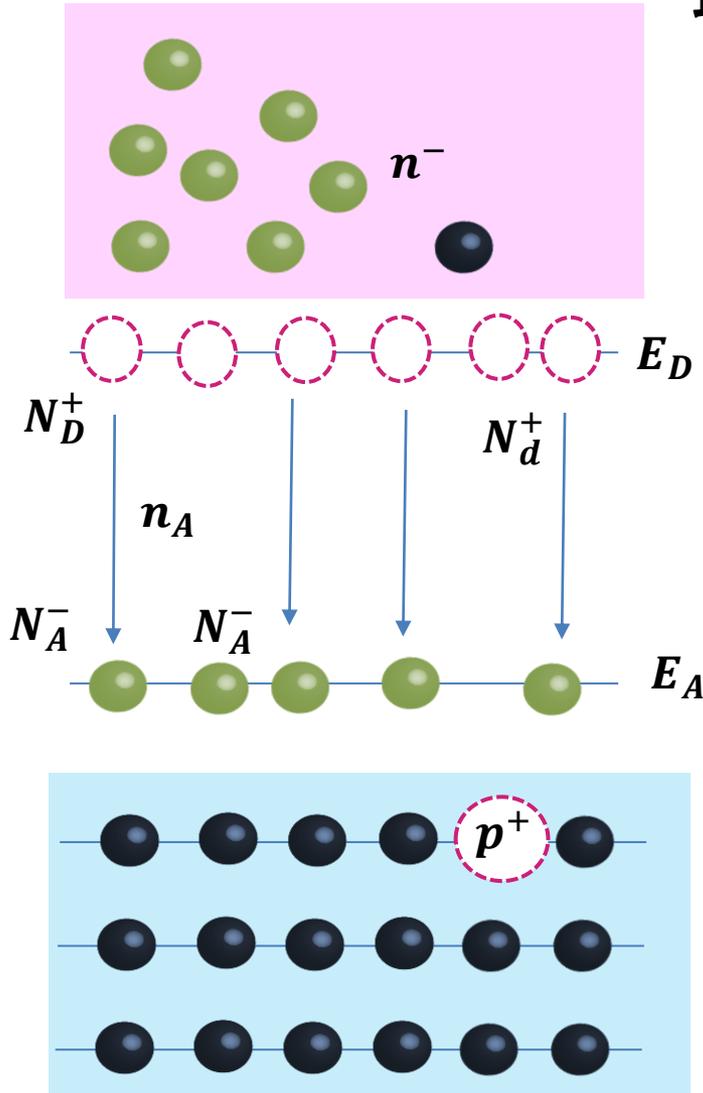
$$p \ll n$$

$$n \cong N_D - N_A$$

$$n \cong N_D - N_A$$

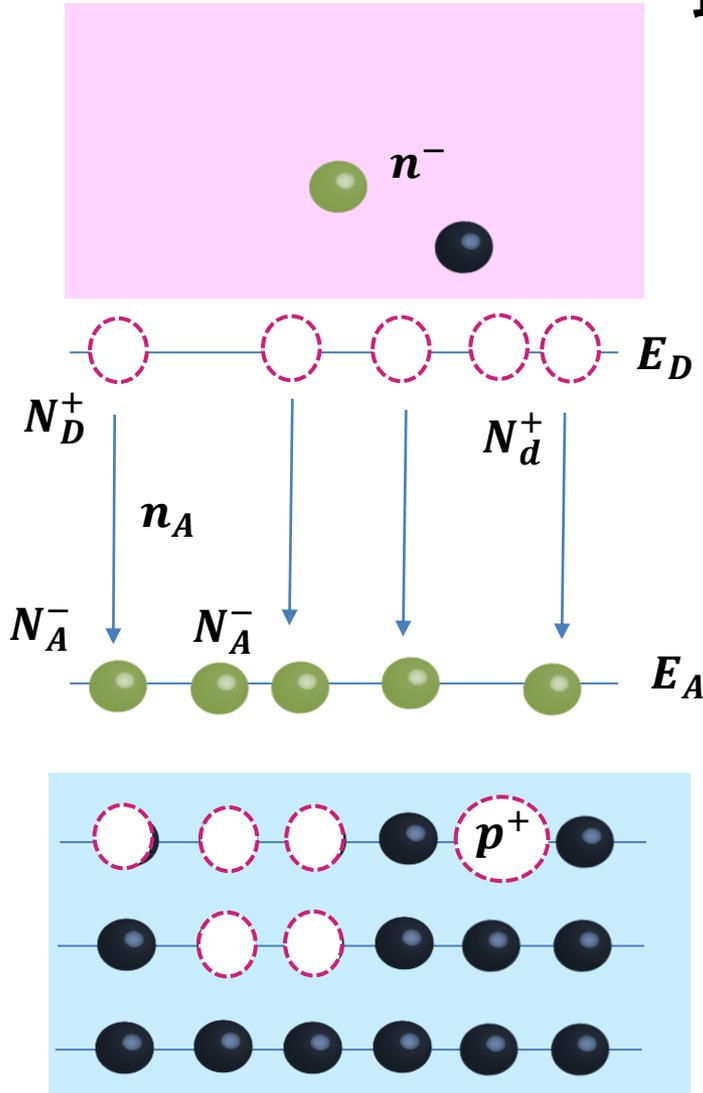
$$p = \frac{n_i^2}{N_D - N_A}$$

半導体の補償効果



P型半導体+少量のN型不純物

最も簡単な場合：完全イオン化近似@室温



$$p^+ + N_D^+ = n^- + N_A^-$$

$$N_A^- = n_A = N_A$$

$$p + N_D = n + N_A$$

$$n \ll p$$

$$p \cong N_A - N_D$$

$$p \cong N_A - N_D$$

$$n = \frac{n_i^2}{N_A - N_D}$$

半導体の補償効果

半導体キャリア密度の温度依存特性: 中級編

N型半導体の電子密度の温度依存特性

$$n = p + N_d^+$$

$$n = N_c \exp\left(\frac{E_f - E_c}{kT}\right) \quad p = N_v \exp\left(\frac{E_v - E_f}{kT}\right)$$

$$N_D^+ = N_D \exp\left(\frac{E_d - E_f}{kT}\right)$$

$$p \ll n$$

$$N_c \exp\left(\frac{E_f - E_c}{kT}\right) = N_D \exp\left(\frac{E_d - E_f}{kT}\right)$$

N型半導体の電子密度の温度依存特性

$$N_c \exp\left(\frac{E_f - E_c}{kT}\right) = N_D \exp\left(\frac{E_d - E_f}{kT}\right)$$

$$E_f = \frac{E_c + E_d}{2} + \frac{1}{2} kT \ln \frac{N_D}{N_c}$$

$$E_f - E_c = \frac{E_d - E_c}{2} + \frac{1}{2} kT \ln \frac{N_D}{N_c}$$

$$n = N_c \exp\left(\frac{\frac{E_d - E_c}{2} + \frac{1}{2} kT \ln \frac{N_D}{N_c}}{kT}\right)$$

N型半導体の電子密度の温度依存特性

$$n = N_c \exp\left(\frac{E_D - E_c + kT \ln \frac{N_D}{N_c}}{2kT}\right)$$

$$n = N_c \exp\left(-\frac{E_c - E_D}{2kT}\right) \exp\left(\frac{kT \ln \frac{N_D}{N_c}}{2kT}\right)$$

$$n = N_c \exp\left(-\frac{E_c - E_D}{2kT}\right) \exp\left(\ln\left(\frac{N_D}{N_c}\right)^{\frac{1}{2}}\right)$$

$$n = N_c \left(\frac{N_D}{N_c}\right)^{\frac{1}{2}} \exp\left(-\frac{E_c - E_D}{2kT}\right)$$

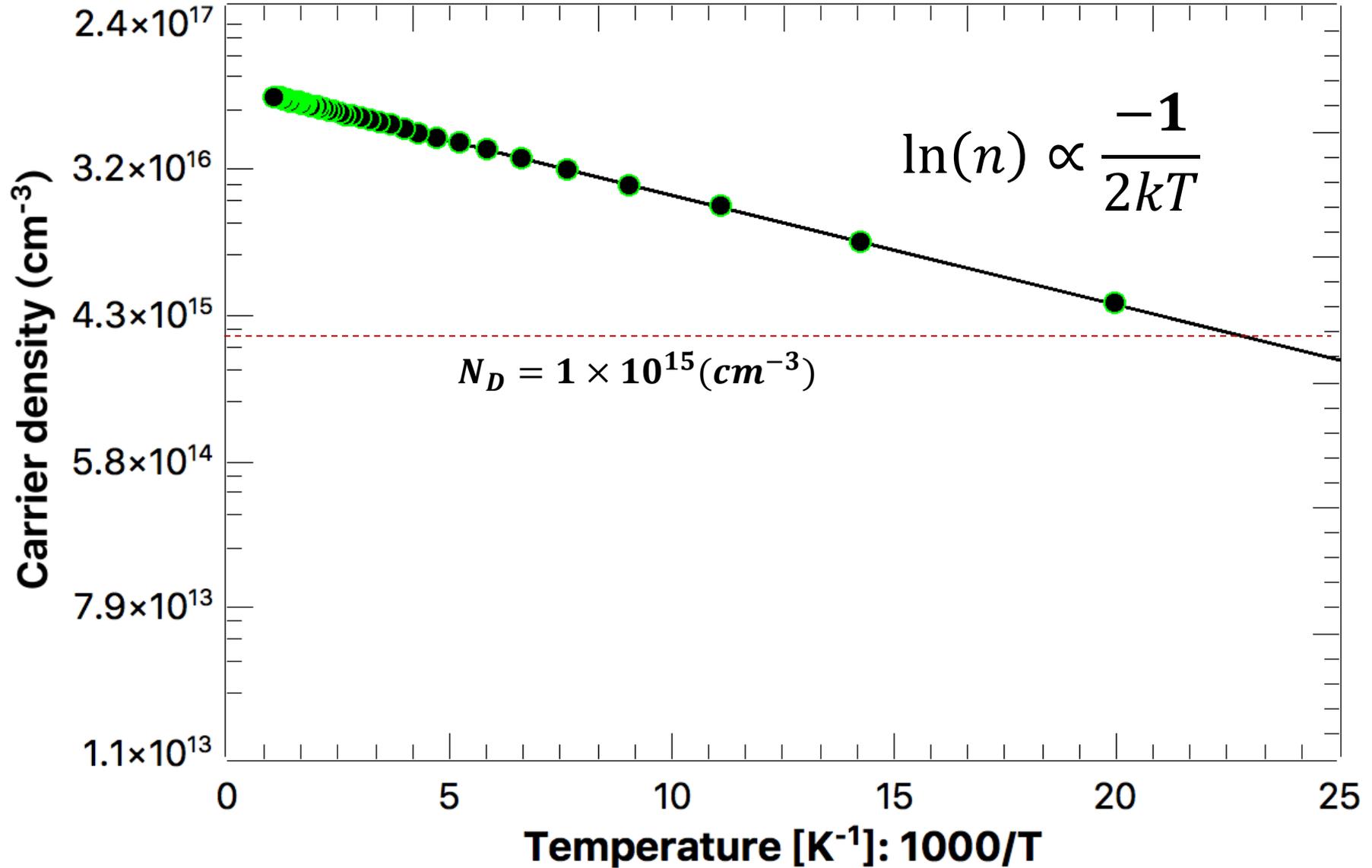
N型半導体の電子密度の温度依存特性

$$n = \sqrt{N_c N_D} \exp\left(-\frac{E_c - E_D}{2kT}\right) \quad p = \frac{n_i^2}{n}$$

$$n = \sqrt{N_c N_D} \exp\left(-\frac{0.025}{2kT}\right)$$

$$\ln(n) \propto \frac{-1}{2kT}$$

N型半導体の電子密度の温度依存特性



N型半導体キャリア密度の温度依存特性: 上級編

$p \ll n$ 近似を使わない $np = n_i^2$ を使う

$$n = p + N_D^+ \quad \longrightarrow \quad p = n - N_D^+$$

$$n(n - N_D^+) = n_i^2$$

$$n^2 - nN_D^+ - n_i^2 = 0$$

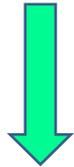
$$n = N_c \exp\left(\frac{E_f - E_c}{kT}\right)$$



$$\exp\left(\frac{E_f}{kT}\right) = \frac{n}{N_c} \exp\left(\frac{E_c}{kT}\right)$$

N型半導体キャリア密度の温度依存特性:

$$N_d^+ = N_d \frac{1}{\exp\left(\frac{E_f - E_D}{kT}\right) + 1}$$



$$\exp\left(\frac{E_f - E_D}{kT}\right) + 1 = \frac{N_D}{N_D^+}$$



半導体キャリア密度の温度依存特

$$\exp\left(\frac{-E_D}{kT}\right) \exp\left(\frac{E_f}{kT}\right) + 1 = \frac{N_D}{N_D^+}$$

$$\exp\left(\frac{E_f}{kT}\right) = \frac{n}{N_C} \exp\left(\frac{E_C}{kT}\right)$$

N型半導体キャリア密度の温度依存特性:

$$\exp\left(\frac{-E_D}{kT}\right) \exp\left(\frac{E_C}{kT}\right) \frac{n}{N_C} + 1 = \frac{N_D}{N_D^+}$$

$$\frac{N_D}{N_D^+} = \exp\left(\frac{E_D}{kT}\right) \frac{n}{N_C} \exp\left(-\frac{E_C}{kT}\right) + 1$$

$$\frac{N_D}{N_D^+} = \frac{n}{N_C} \exp\left(\frac{E_C - E_D}{kT}\right) + 1$$

N型半導体キャリア密度の温度依存特性:

$$E_C - E_D = 0.025[eV]$$

$$\frac{N_D}{N_D^+} = \frac{n}{N_C} \exp\left(\frac{0.025}{kT}\right) + 1 \approx \frac{N_D}{N_C} \exp\left(\frac{0.025}{kT}\right) + 1 = B$$

$$N_D^+ = \frac{N_D}{B}$$

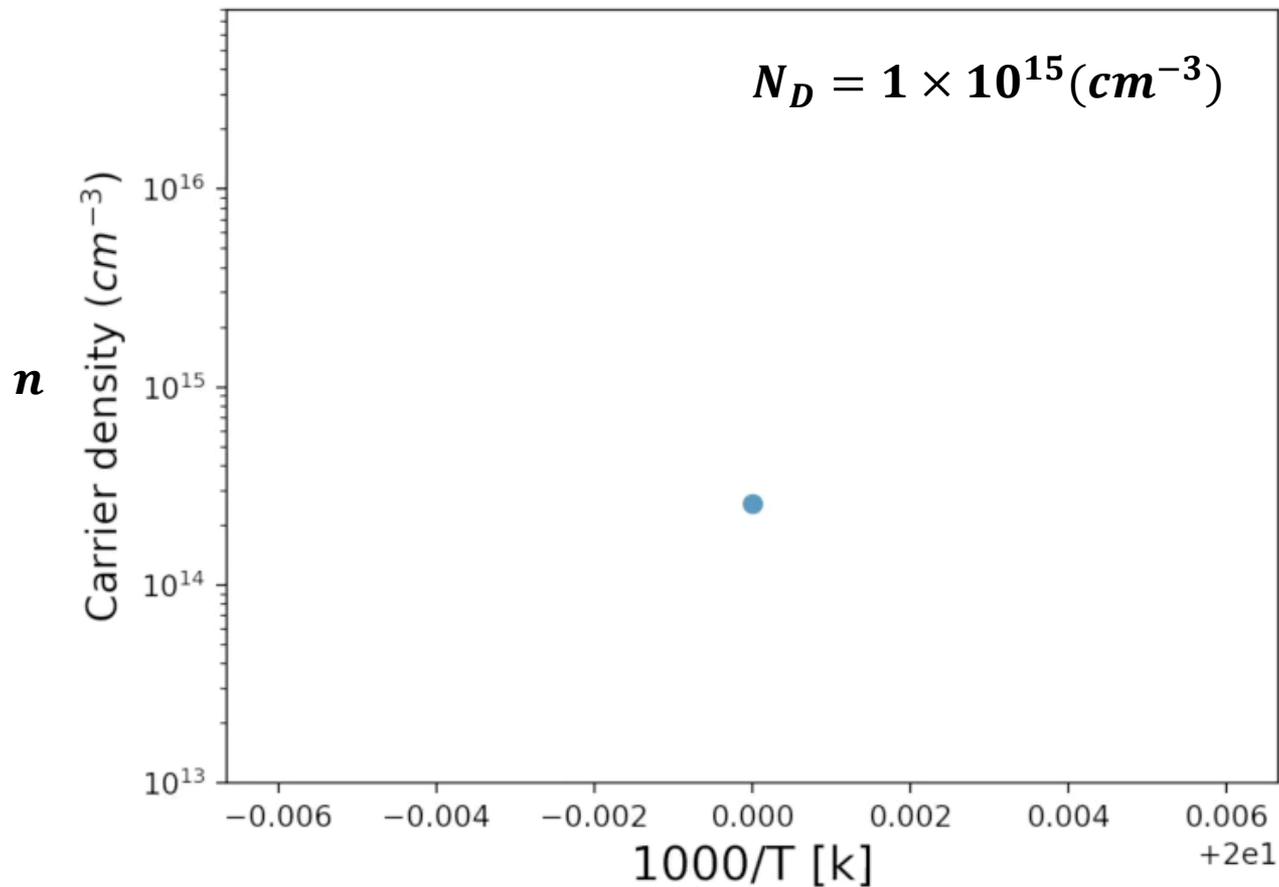
$$n^2 - n \frac{N_D}{B} - n_i^2 = 0$$

$$Bn^2 - nN_D - Bn_i^2 = 0$$

N型半導体キャリア密度の温度依存特性:

$$n = \frac{N_D + \sqrt{N_D^2 + 4(B)^2 n_i^2}}{2B}$$

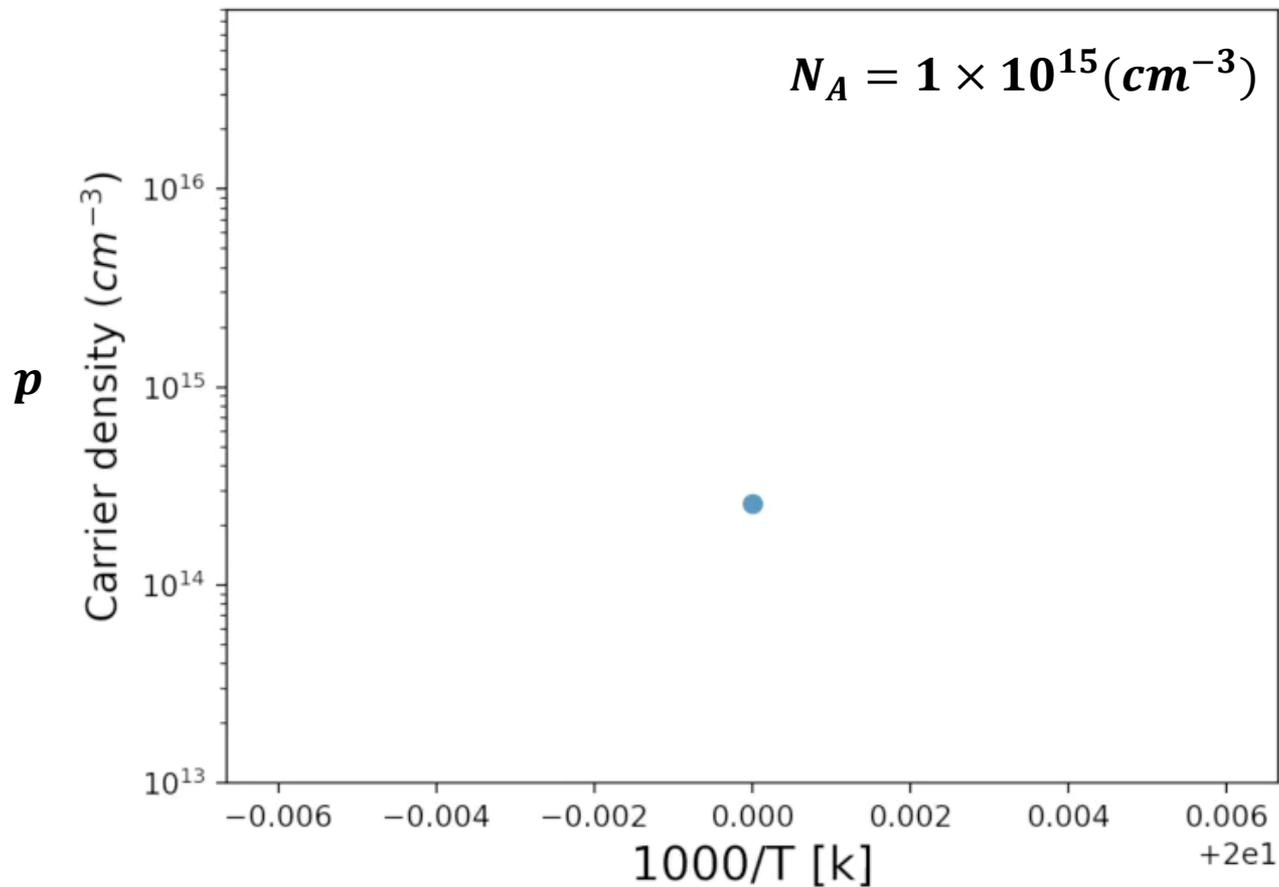
$$B = \frac{N_D}{N_C} \exp\left(\frac{0.025}{kT}\right) + 1$$



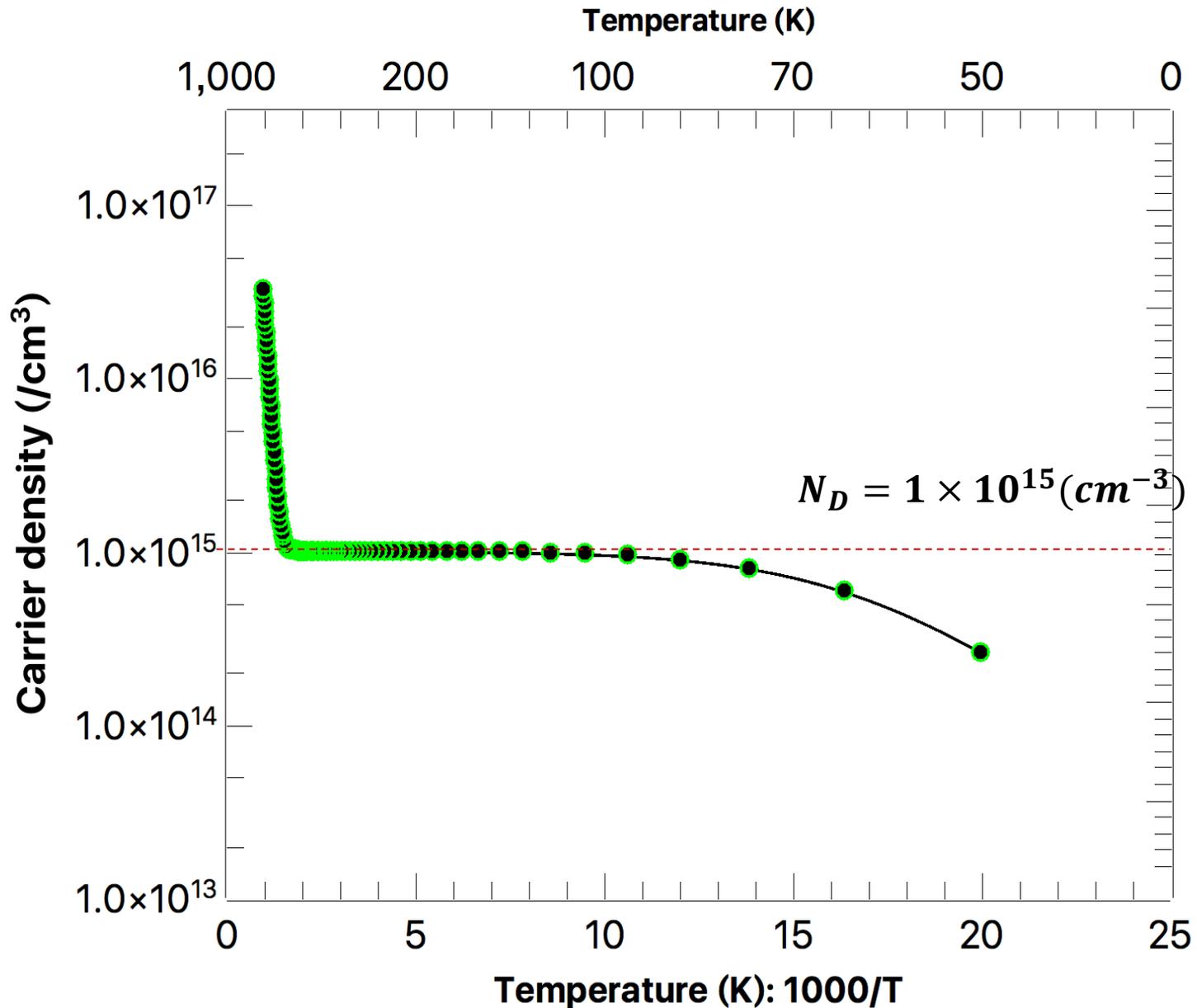
P型半導体キャリア密度の温度依存特性:

$$p = \frac{N_A + \sqrt{N_A^2 + 4(B)^2 n_i^2}}{2C}$$

$$\frac{N_A}{N_V} \exp\left(\frac{0.025}{kT}\right) + 1 = C$$



N型半導体キャリア密度の温度依存特性:



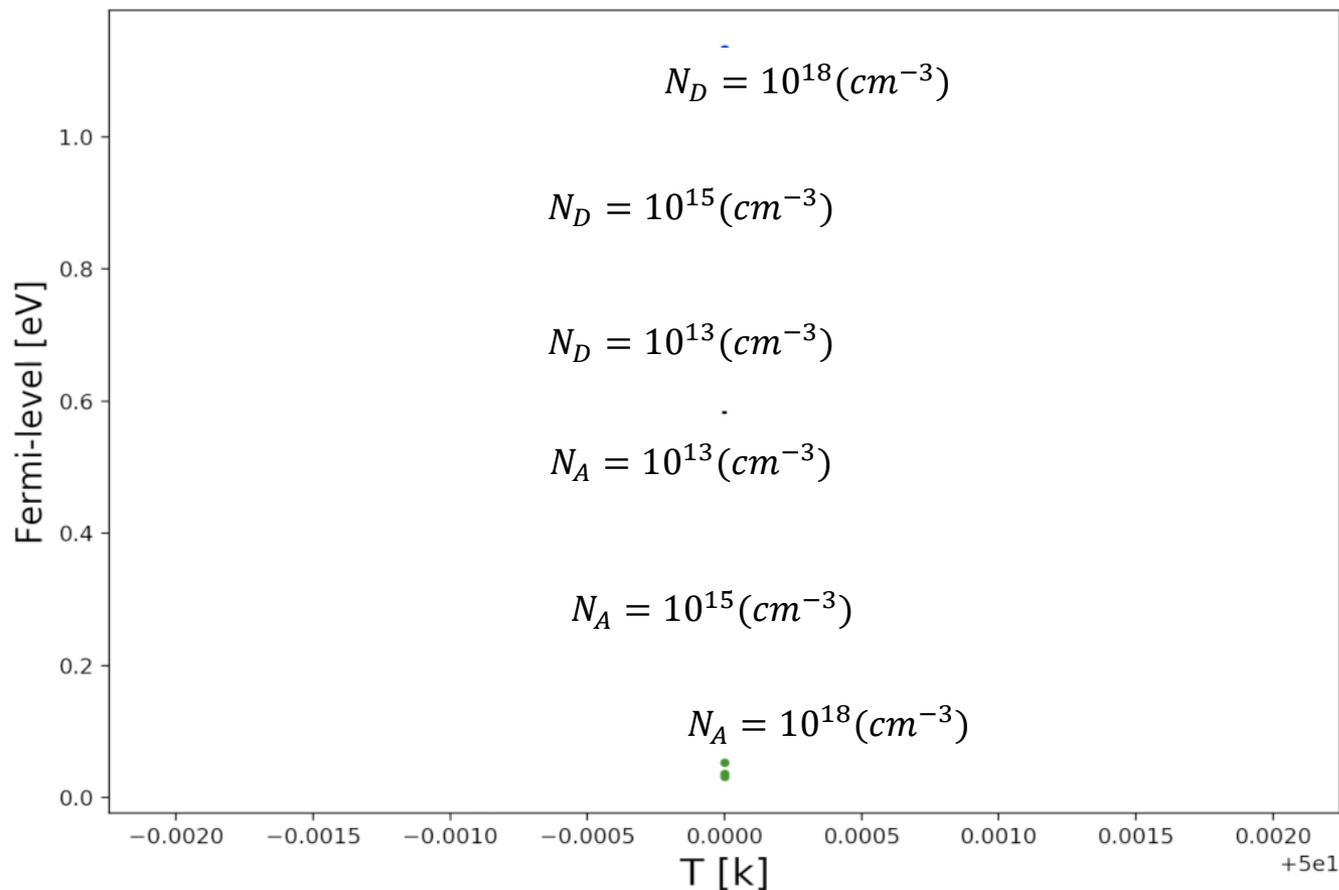
P型半導体キャリア密度の温度依存特性:

$$n = N_c \exp\left(\frac{E_f - E_c}{kT}\right)$$

$$E_{fn} = E_c + kT \ln\left(\frac{n}{N_c}\right)$$

$$p = N_v \exp\left(\frac{E_v - E_f}{kT}\right)$$

$$E_{fp} = E_v - kT \ln\left(\frac{p}{N_v}\right)$$



N型/P型半導体のフェルミ準位の温度依存特性:

